PESSIMISM, DISAGREEMENT, AND ECONOMIC FLUCTUATIONS

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Abstract
The pessimistic bias and the cross-sectional dispersion of households' subjective beliefs heighten during recessions. We provide empirical evidence for a dominant non-inflationary aggregate demand shock that accounts for the bulk of business-cycle fluctuations not only in real quantities but also in (1) pessimism—to what degree households are more pessimistic than the rational expectation benchmark and (2) disagreement—the cross-sectional dispersion of households' beliefs. To rationalize the empirical findings, this paper develops a theory of ambiguity-driven business cycles, where the Bayesian formulation of the ambiguity shock can generate positive co-movements across real quantities together with counter-cyclical pessimism and disagreement within the real business-cycle framework. Our theory reproduces the salient features of the business cycles extended with survey data on households' expectations. Quantitatively, the ambiguity shock alone accounts for a significant fraction of the business-cycle fluctuations in pessimism, disagreement, and real quantities. (JEL: E32, E13, D8)

1. Introduction

When forming expectations about the outlook of the economy, households are more pessimistic than what is implied by the rational expectations and they disagree with each other. Using survey data on households' expectations, we can empirically quantify

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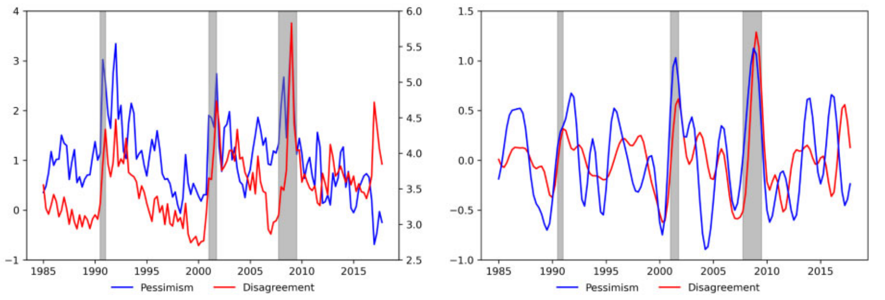

(a) raw data (b) bandpass filtered data

FIGURE 1. Pessimism and disagreement over the business cycles. The figure plots pessimism (blue line) and disagreement (red line) over the period of 1985:I–2017:IV. Panel (a) plots the raw time series of pessimism (left-hand axis) and disagreement (right-hand axis). Panel (b) plots the cyclical components of pessimism and disagreement, bandpass-filtered at frequencies of 6–32 quarters. The correlation between pessimism and disagreement is 0.43 for the raw data and 0.50 for the band-pass filtered data. Moreover, for the bandpass filtered data, the correlations of output with pessimism and disagreement are −0.45 and −0.71, respectively. Shaded areas correspond to NBER recessions. Time series of pessimism and disagreement are constructed following Bhandari, Borovicka, and Ho (2019).

(1) pessimism by the distance between households' subjective beliefs and rational expectations and (2) disagreement by the cross-sectional dispersion of households' beliefs.[1] In the data, both two display a considerable amount of business-cycle volatility and counter-cyclical fluctuations. Figure 1 plots the raw and the bandpass filtered time-series of pessimism and disagreement over the period of 1985:I–2017:IV. Recessions are times with heightened pessimism and disagreement among households.

The empirical relevancy of the business-cycle fluctuations in households' subjective beliefs can be further explored by unveiling the dynamic co-movements across pessimism, disagreement, and real quantities. We demonstrate that business cycles admit a dominant non-inflationary aggregate demand shock, which can be originated from business-cycle variations in pessimism and disagreement. This paper proceeds to develop a theory of ambiguity-driven business cycles, in which a Bayesian formulation of the ambiguity shock generates empirically relevant business-cycle fluctuations and dynamic co-movements in pessimism, disagreement, and real quantities. Quantitatively, the ambiguity shock serves as the dominant non-inflationary aggregate demand shock in driving the business-cycle fluctuations not only in real quantities but also pessimism and disagreement among households.

Empirical Analysis. We estimate a VAR on a set of macroeconomic variables, which covers both real and nominal sides of the economy together with two additional time series characterizing households' subjective beliefs: (1) pessimism (P), which measures to what extent households over-estimate unemployment rate in the next

1. Time series of pessimism and disagreement are constructed following Bhandari, Borovicka, and Ho (2019).

12 months relative to professional forecasters, and (2) disagreement (D), which measures the cross-sectional dispersion of households' beliefs on unemployment rate in the next 12 months, over the period 1985:I–2017:IV.

In our baseline specification, we identify an structural vector autoregression (SVAR)-based shock that accounts for the maximum volatility in both pessimism (P) and disagreement (D) and name it the PD shock. Over the business-cycle frequencies, the identified PD shock explains about 70% of volatility in pessimism, about 60% of volatility in disagreement, and about 40% of volatility in unemployment, output, hours worked, and investment. The PD shock is disconnected from the long-run variations in the data: it accounts for less than 10% volatility in all macroeconomic variables, including pessimism and disagreement. Most importantly, the PD shock is disconnected from either total factor productivity (TFP) or inflation at all frequencies: it explains less than 5% volatility in TFP and inflation either in the short or in the long run. Moreover, the impulse response functions (IRFs) of all variables to the identified PD shock suggest that the PD shock produces non-inflationary demand-driven business cycles: real quantities, including unemployment, output, hours worked, investment, and consumption co-move positively without commensurate movements in TFP or inflation. At the same time, the PD shock also gives rise to counter-cyclical pessimism and disagreement. Similar pictures emerge when we identify an SVAR-based shock that targets any one of the real quantities, including output, investment, hours worked, and investment. The set of SVAR-based shocks is interchangeable in the sense that they give rise to almost identical IRFs, highly correlated conditional time series, and similar variance contributions.

Empirically, the data admit a dominant shock[2] that drives the bulk of the business-cycle fluctuations not only for real quantities but also for pessimism and disagreement among households. The dominant shock creates positive co-movements across real quantities and counter-cyclical pessimism and disagreement. It accounts for little volatility in the long run. Importantly, it is disconnected from either TFP or inflation at all frequencies. In other words, macroeconomic data extended with survey data on households' expectations suggest the existence of a dominant non-inflationary aggregate demand shock or propagation mechanism that drives the dynamic co-movements and the bulk of the fluctuations among pessimism, disagreement, and real quantities at the business-cycle frequencies.

A Theory of Ambiguity-Driven Business Cycles. We proceed to develop a theory of ambiguity-driven business cycles to capture the above salient features of the macroeconomic data extended with survey data on households' expectations.

2. We follow Angeletos, Collard, and Dellas (2020) in interchanging shocks and propagation mechanisms without loss of generality in the empirical analysis. When bringing the empirical findings to the theory, we take a stance by interpreting the identified empirical object as an exogenous structural shock instead of the common propagation mechanism of many other structural shocks. Section 3.3 discusses to what extent the common-propagation-multiple-shock interpretation is (in)consistent with existing literature.

Our theory deviates from rational expectations by the introduction of ambiguity averse decision-makers (DMs) who perceive ambiguity[3] about the aggregate fundamental. With ambiguity averse preferences represented by the (recursive) smooth model (Klibanoff, Marinacci, and Mukerji 2005, 2009), we provide a Bayesian formulation of the ambiguity shock, which varies the amount of ambiguity perceived by DMs without changing their preferences. More importantly, it enables our theory to provide novel insights into the counter-cyclical co-movements in pessimism and disagreement when coupled with incomplete information. Specifically, in response to an adverse ambiguity shock that makes DMs more ambiguous about the set of possible models in their priors, they behave as if they perceive that the aggregate fundamental is weaker and more volatile. The former transforms into a deterioration in the degree of households' pessimism. The latter increases the reliance on private information when agents form expectations since the quality of prior information gets worse, which eventually transforms into a higher cross-sectional dispersion of beliefs, namely larger disagreement. We call it the dual impacts of the ambiguity shock, which are at the core of explaining the co-movements between pessimism and disagreement.

We embed this decision-making framework into an otherwise standard real business-cycle (RBC) model[4] that features (a) aggregate demand externalities and (b) incomplete information about aggregate fundamentals. We start with a static RBC model that abstracts out capital accumulation and show that an adverse ambiguity shock generates a recession with heightened pessimism and disagreement. At the core of the result is the interplay between incomplete information and the dual impacts of the ambiguity shock, which is analyzed through a game-theoretic interpretation of the market equilibrium. The model features multiple islands differing in productivity that consist of an aggregate and an idiosyncratic component. Local DMs, including firms and workers, have perfect information about local productivity but incomplete information about the aggregate productivity of the economy. When local DMs make productive factors demand and supply decisions, they form expectations about the aggregate productivity since it is the sufficient statistics of the demand or the terms of trade for local commodities in the general equilibrium. Ambiguity averse DMs possess ambiguity about aggregate productivity. Therefore, they are pessimistic about the economy: they perceive models with weaker demand to be more likely than those with stronger demand. In response to an adverse ambiguity shock, the local DMs become more pessimistic about the demand for local commodities. As a consequence, island production shrinks without any commensurate movement in factor productivity. At the aggregate level, the economy plummets as if there had been a contractionary aggregate demand shock. The ambiguity shock propagates as if it was the confidence shock á la Angeletos, Collard, and Dellas (2018). Whereas, the ambiguity shock can additionally induce empirically relevant counter-cyclical co-movements in pessimism and disagreement.

3. According to Marinacci (2015), ambiguity refers to subjective uncertainty over probabilities due to lack of ex-ante information to pin down a specific model for the economy in the course of decision-making.

4. The model can, otherwise, be regarded as the flexible price benchmark of the standard NK model.

Finally, we quantitatively evaluate the impacts of the ambiguity shock through the lens of a dynamic stochastic general equilibrium (DSGE) model that features a rich set of "bells-and-whistles", including habit formation, investment adjustment cost, and variable capital utilization, but with flexible prices. The ambiguity shock in the DSGE model serves as the theoretical counterpart to the empirically identified PD shock, which can be justified by the fact that the ambiguity shock is the only type of shock that can generate positive co-movements in pessimism and disagreement. We estimate the model by matching the IRFs of the PD shock and those of the ambiguity shock. The estimated model successfully captures the dynamic co-movements in pessimism, disagreement, and real quantities, which hinges on the nature of the ambiguity shock being the non-inflationary aggregate demand shock. Moreover, our quantitative DSGE model can reproduce the "interchangeability" as identified in our empirical analysis. And the ambiguity shock alone captures most of the volatility in pessimism, disagreement, and real quantities at the business-cycle frequencies. It then implies that ambiguity shock resembles the dominant non-inflationary aggregate demand shock identified in our empirical analysis. Our DSGE model successfully reproduces the salient features in the macroeconomic data extended with survey data on households' expectations.

Contributions. Empirically, the paper contributes to the literature that studies deviations from full-information rational expectations. We unveil the patterns of the dynamic co-movements across macroeconomic quantities and households' subjective beliefs, providing evidence of the fact that households' subjective beliefs have non-trivial roles in driving the business cycles.

Theoretically, the paper contributes to the literature with a Bayesian formulation of the ambiguity shock by embedding the smooth model of ambiguity into general equilibrium macroeconomic models. The unique insight of our Bayesian formulation of the ambiguity shock that cannot be shared with others is that: when coupled with incomplete information, the ambiguity shock can explain business-cycle fluctuations among pessimism, disagreement, and real quantities without relying on the new Keynesian Phillips curve (NKPC) as the propagation mechanism.[5]

Layout. The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 reports the empirical findings of our paper. Section 4 sets up the static model without capital. Section 5 characterizes the equilibrium of the static model and studies the impacts of the ambiguity shock. Section 6 demonstrates the quantitative potential of our theory through the lens of an estimated DSGE model. Finally, Section 7 concludes the paper.

5. Incomplete information is important because it breaks the Barro–King critique by moving labor choices prior to the common knowledge of aggregate productivity as in Ilut and Saijo (2021) and Angeletos and Lian (2021), which is the primary reason why ambiguity shock is capable of serving as the dominant non-inflationary aggregate demand shock in driving business-cycle fluctuations and dynamic co-movements among pessimism, disagreement, and real quantities, a certain feature of the data that has been confirmed in our empirical analysis.

2. Related Literature

The empirical analysis of the paper builds on the "multiple-cuts" strategies of Angeletos, Collard, and Dellas (2020). In their paper, they find that data on macroeconomic variables admits a dominant non-inflationary aggregate demand shock to account for the bulk of the business-cycle fluctuations in real quantities, a certain feature they call "anatomy". We complement their findings by providing empirical evidence suggesting that the "anatomy" can be linked to households' subjective beliefs characterized by pessimism and disagreement. We follow Bhandari, Borovicka, and Ho (2019) in the construction of time series for pessimism and disagreement. In their paper, the authors construct the same set of time series not only for unemployment rate forecasts but also for inflation forecasts and focus on the dynamic co-movements across the pessimism or belief wedges in unemployment and inflation. Our paper complements their empirical findings by providing additional evidence on the dynamic co-movement patterns across pessimism, disagreement, and real quantities.

Our paper is closely related to Ilut and Schneider (2014) and Bhandari, Borovicka, and Ho (2019), both of which study the implications of ambiguity and ambiguity aversion in developing an exotic shock that drives the business cycles. Our paper differs from these two papers in a few respects. Ilut and Schneider (2014) use the multiple priors preferences axiomatized by Gilboa and Schmeidler (1989) to model ambiguity aversion, and the notion of the ambiguity shock is in a classical statistics fashion. Bhandari, Borovicka, and Ho (2019) use the robust preference proposed by Hansen and Sargent (2001a,b) and focus on time-varying concerns for model misspecification, which can be understood as time-variations in the degree of ambiguity aversion. In our paper, ambiguity averse preference is represented by the (recursive) smooth model of ambiguity axiomatized by Klibanoff, Marinacci, and Mukerji (2005, 2009) and the learning process follows the smooth rule of updating proposed by Hanany and Klibanoff (2009) to ensure dynamic consistency. Conceptually, our Bayesian formulation of ambiguity shock differs from both works in the sense that it is a shock to the amount of ambiguity rather than a shock to DMs' taste for ambiguity (Bhandari, Borovicka, and Ho 2019) or a mix of both (Ilut and Schneider 2014). Furthermore, in both of the two works, the shocks of interest are formulated in the news- or noise-like fashion, which relies on the NKPC as the propagation mechanism to generate positive co-movements across real quantities. However, in our paper, the ambiguity shock propagates in a similar fashion to the confidence shock, which generates demand-driven aggregate fluctuations without nominal rigidity. Finally, Bhandari, Borovicka, and Ho (2019) provide no implications about variations in the cross-sectional dispersion of beliefs, or disagreement in the terminology of our paper. Instead of considering disagreement for DMs within the model, Ilut and Schneider (2014) focus on the disagreement across professional forecasters, who are agents outside the model. Ilut and Schneider (2014) treat the disagreement across professional forecasters as the observable counterparts of the ambiguity process and use it to discipline the variations in the ambiguity. In our paper, we provide empirical evidence that the business-cycle variations in disagreement across households originate from ambiguity shock. Our theory then treats it as an endogenous variable, whose business-cycle variations arise endogenously due to the ambiguity shock.

Ilut and Saijo (2021) is an exception in the literature that studies the business-cycle implications of ambiguity and ambiguity aversion. Instead of developing an exotic shock based on ambiguity averse preference, the authors develop a theory, where ambiguity aversion serves as an endogenous propagation mechanism of business cycles. In principle, they can also generate demand-driven business cycles without relying on NKPC as the propagation mechanism. From this perspective, the authors provide an alternative framework for rationalizing the empirical findings of our paper. In addition to interpreting our empirical findings as the result of an exotic structural shock, it can also be interpreted as a common propagation mechanism of multiple structural shocks. However, on the same methodological ground as Ilut and Schneider (2014), there is no explanation for the business-cycle fluctuations of disagreement across households, who are DMs within the model.[6]

Our paper connects to the theory of the confidence shock á la Angeletos and La'O (2013), Angeletos, Collard, and Dellas (2018), Huo and Takayama (2015), and Benhabib, Wang, and Wen (2015) in generating animal-spirit-like demand-driven aggregate fluctuations. Even though the ambiguity shock propagates to fluctuations in real quantities in a similar fashion to the confidence shock, in our theory, it additionally creates empirically relevant counter-cyclical variations in pessimism and disagreement. The latter is the unique feature of our theory when making a comparison with the confidence shock literature. In a broader context, our paper is also related to (a) the news shock literature, Beaudry and Portier (2004, 2006) and Jaimovich and Rebelo (2009) and (b) the noise shock literature, Lorenzoni (2009), Barsky and Sims (2012), and Blanchard, L'Huillier, and Lorenzoni (2013) in arguing that business cycles are expectation-driven.

Counter-cyclical disagreement or cross-sectional dispersion of beliefs can alternatively be explained by the uncertainty or risk shock literature. However, either they cannot generate demand-driven business cycles (Bloom 2009 and Bloom et al. 2018) or they provide no implications about the business-cycle fluctuations in pessimism (Bloom 2009; Bloom et al. 2018; and Arellano, Bai, and Kehoe 2018). A notable exception is Bidder and Smith (2012), in which the risk shock can generate variations in pessimism and demand-driven fluctuations in real quantities when households have robust preferences and Jaimovich–Rebelo-type utility. However, in their paper, the bulk of the business-cycle fluctuations is due to the TFP shock. The risk shock only explains a tiny amount of business-cycle fluctuations in the data, which is inconsistent with our empirical findings.

Ambiguity averse preferences have been intensively used in the literature to generate asymmetric responses of the economy to shocks in recessions or booms; see, for example, Epstein and Schneider (2008), Ilut (2012), Ilut, Kehrig, and Schneider (2018), Baqaee (2020), and Zhang (2022) in the context of multiple prior preferences and Chen et al. (2022) in the context of smooth model. These works focus on ambiguity

6. Whereas, we acknowledge that Ilut and Saijo (2021) can provide an alternative interpretation of our empirical results upon extending the framework with dispersed information. Section 3.3 provides a detailed discussion.

about the precisions of fundamentals or signals, which makes the economy respond more strongly to shocks in recessions than in booms. In our paper, ambiguity is about the mean of the fundamental, but the ambiguity shock itself is of second-moment. Therefore, we also have the implication of counter-cyclical responses to the aggregate productivity shock, though the reasoning is different. Finally, the role of ambiguity and ambiguity aversion has also been studied in the asset pricing literature such as Ju and Miao (2012), Bianchi, Ilut, and Schneider (2017), and Collard et al. (2018).

Finally, our paper also relates to the theory of the beauty contest (Morris and Shin 2002; Angeletos and Pavan 2007) and those works study coordination games with model uncertainty (Chen, Lu, and Suen 2016 and Chen and Suen 2016).

3. The Empirical Analysis

This section conducts an empirical analysis to highlight the empirical relevancy of the business-cycle variations in pessimism and disagreement.

3.1. Data and Method

Our empirical analysis extends that in Angeletos, Collard, and Dellas (2020) with two additional macroeconomic time series that measure pessimism and disagreement respectively, both of which are constructed in the same way as in Bhandari, Borovicka, and Ho (2019). Specifically, we measure pessimism (P) by the gap between the average of households' year-ahead unemployment rate forecasts from the Michigan Survey of Consumers and its counterparts in the Survey of Professional Forecasters, which proxies the rational expectation benchmark. At the same time, we measure disagreement (D) by the cross-sectional dispersion of the households' year-ahead unemployment rate forecasts.[7] Therefore, in our specification, the data consists of twelve macroeconomic variables: real gross domestic product (GDP) per capita (Y); investment (I); consumption (C); hours worked per person (h); unemployment rate (UNEMP); the labor productivity (Y/h); the level of utilization-adjusted TFP; the labor share (wh/Y); the inflation rate (π); the federal funds rate (R); pessimism (P); and Disagreement (D), ranging from 1985:I to 2017:IV.[8] Details of definitions and sources of data can be found in Online Appendix B. In what follows, we briefly discuss the empirical strategy used in our identification, which has been extensively discussed in Angeletos, Collard, and Dellas (2020).

We start by estimating a vector autoregression (VAR) of the form:

$$A(L)X_t = \nu_t,$$

7. See Online Appendix C of Bhandari, Borovicka, and Ho (2019) for the details in the construction of these two measures.

8. We have access to the survey data in the Michigan Survey of Consumer from 1982:I. We drop the first three years because disagreement seems to have decreased systematically ever since around 1985, which corresponds to the start of the Great Moderation. Note that our empirical results are robust to the current sample selection. The same image appears even if we start from 1982:I.

where X_t corresponds to the data matrix described above and $A(L) \equiv \sum_{r=0}^{p} A_r L^r$ is the matrix polynomials in the lag-operator L with $A_0 = I$. In our baseline specification, the VAR is estimated in the Bayesian method with Minnesota prior and the number of lags P is set to be 2 following the Akaike information criterion. We then recover a set of SVAR-based shocks, each of which is a linear combination of VAR residuals. Assume that there exists a mapping between VAR residuals ν_t and mutually independent shocks ε_t with identity variance–covariance matrix $\mathbb{E}[\varepsilon_t \varepsilon_t'] = I$:

$$\nu_t = \widetilde{S} Q \varepsilon_t,$$

where $\widetilde{S}$ is the Cholesky decomposition of $\Sigma \equiv \mathbb{E}[\nu_t \nu_t']$ and Q is some orthonormal matrix, that is, $Q^{-1} = Q'$. Additional restrictions over Q are needed for the identification of SVAR-based shocks ε_t. In our specification, we implement the max-share strategy in the frequency domain. Specifically, we recover a set of SVAR-based shocks, the identification of which is equivalent to choosing a column vector q that maximizes the contribution of the shock to the entire volatility of a variable of interest or a combination of them over a certain frequency band in the frequency domain:

$$q^* = \arg\max_q q' \Theta(k, \underline{\omega}, \overline{\omega}) q,$$

where matrix $\Theta(k, \underline{\omega}, \overline{\omega})$ denotes the entire volatility of the kth variable[9] in data X_t over the frequency band $[\underline{\omega}, \overline{\omega}]$ and can be directly computed using VAR residuals.[10] We focus on the business-cycle frequencies. Therefore, following Stock and Watson (1999), we set the frequency band to $[\underline{\omega}, \overline{\omega}] = [2\pi/32, 2\pi/6]$. Our empirical results consist of a set of identified SVAR-based shocks obtained by varying the targeted variable(s) of interest together with their empirical properties, such as variance contributions and IRFs.

3.2. *The Empirical Results*

This subsection presents our empirical findings. We start by demonstrating the empirical properties of our baseline PD shock, which is identified by jointly targeting pessimism and disagreement. The joint explanatory power of the PD shock allows us to map it to the ambiguity shock in our theoretical model later on. We then move on to demonstrate the interchangeability among the PD shock and his many "cousin" shocks. We close up this subsection with a summary of our empirical results, in which the identified PD shock is interpreted as a structural shock. However, according to Cochrane (1994), shocks and propagation mechanisms cannot be separated. Therefore, the identified PD shock can also be interpreted as the footprints of many other structural shocks through a common propagation mechanism. Section 3.3 discusses to what extent the common-propagation-multiple-shock interpretation is (in)consistent with existing literature of business-cycle theories.

9. The choice of k may also refer to a subset of variables in data X_t, in which case the targeted volatility would be the entire volatility of all variables in the subset k.

10. See Section I of Angeletos, Collard, and Dellas (2020) for more detailed derivations.

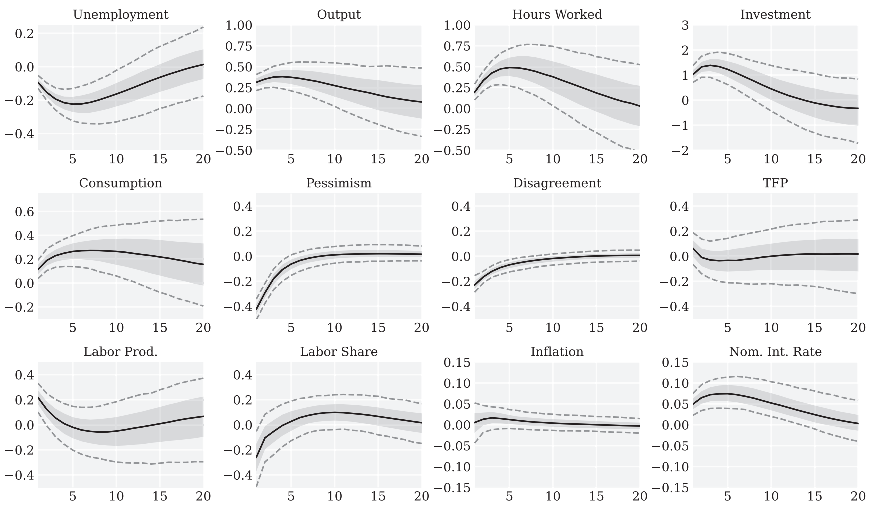

FIGURE 2. Impulse response functions to The PD shock. This figure plots the impulse response functions of all variables to the identified PD shock that jointly targets the volatility of pessimism and disagreement. Horizontal axis: time horizon in quarters. Shaded area: 68% Highest Posterior Density Interval (HPDI). Dashed lines: 95% HPDI.

TABLE 1. Variance contributions of the identified PD shock.

	Pessimism	Disagreement	UNEMP	Y	h	I	C
Short run (6–32 Qrts)	67.30 [58.46, 74.93]	58.15 [47.74, 66.64]	42.32 [31.90, 52.08]	42.08 [32.10, 50.99]	38.42 [28.79, 48.81]	42.18 [32.30, 51.55]	23.12 [15.99, 31.94]
Long run (80–∞ Qrts)	16.97 [4.97, 37.11]	19.46 [5.81, 42.85]	7.96 [2.34,20.43]	5.81 [1.09,18.76]	6.78 [2.09,17.42]	5.70 [1.37,16.37]	5.69 [0.91,19.02]
			TFP	Y/h	wh/Y	π	R
Short run (6–32 Qrts)			3.34 [1.20, 7.69]	12.03 [6.41,19.52]	11.11 [5.25,19.84]	3.84 [1.33, 8.66]	27.12 [16.40,38.14]
Long run (80–∞ Qrts)			4.61 [0.98,15.26]	4.93 [0.86,17.05]	5.36 [1.46,15.21]	5.34 [1.71,15.09]	10.78 [3.58,23.43]

Note: Variance contributions of the identified PD shock at the short-run and long-run frequency bands. The short run corresponds to the frequency band $[\underline{\omega}, \bar{\omega}] = [2\pi/32, 2\pi/6]$. The long run corresponds to the frequency band $[\underline{\omega}, \bar{\omega}] = [0, 2\pi/80]$. The shock is constructed by jointly targeting volatility of pessimism and disagreement at the short-run frequency band. The 68% HPDI are reported in brackets. Disagreement is measured by the cross-sectional dispersion of belief.

The PD Shock. In our baseline setup, we jointly target the volatility of pessimism and disagreement and name the identified shock as the PD shock. We demonstrate the empirical properties of the identified PD shock by looking at (1) the IRFs of all macroeconomic variables to it and (2) its variance contributions to all macroeconomic variables.

Figure 2 plots the empirical IRFs of all macroeconomic variables to the identified PD shock and Table 1 reports the variance contributions of the identified PD shock to all macroeconomic variables. The identified PD shock explains about 70% of volatility in pessimism and about 60% of volatility in disagreement over the business-cycle frequencies. At the same time, the PD shock explains almost 40% of volatility in unemployment, output, hours, and investment. The PD shock is able to create "realistic" business cycles, where all real quantities, including unemployment, output, hours worked, investment, and consumption positively co-move. Notably, it also gives rise to counter-cyclical pessimism and disagreement. Over the business-cycle frequencies, it only explains less than 4% of volatility in TFP and only about 13% of volatility in labor productivity. It suggests that the identified PD shock cannot be mapped to either the canonical TFP shock in the RBC model or the uncertainty shock in Bloom (2009) and Bloom et al. (2018), where the latter gives rise to business-cycle co-movements in real quantities primarily by endogenous variations in productivity. The mild pro-cyclical movements in labor productivity may originate from variable factor utilization, an issue we will address in our quantitative DSGE model. Furthermore, the identified PD shock only explains less than 5% of volatility in inflation, which suggests that the propagation mechanism of the PD shock is orthogonal to the NKPC. The disconnection of the identified PD shock with either productivity or inflation opens up the possibility of interpreting the PD shock as a non-inflationary aggregate demand shock, a certain feature that our theoretical and quantitative DSGE models target to replicate.

Finally, though the identified PD shock explains the bulk of business-cycle fluctuations in pessimism, disagreement, and real quantities in the short run, it has a negligible impact in the long run: it can only explain less than 10% volatility of macroeconomic variables are explained and less than 20% volatility in pessimism and disagreement. A similar image emerges from the empirical IRFs: (1) the IRFs of real quantities, including unemployment (UNEMP), output (Y), hours (h), investment (I), and consumption (C), peak at around 5 quarters and fade away quickly after 15–20 quarters; and (2) the IRFs of pessimism and disagreement are monotonically increasing and fade away after 15–20 quarters.

Interchangeability. We move on to compare our PD shock with its many "cousin" shocks, each of which targets the volatility of other real quantities: GDP, investment, hours, unemployment, and consumption one at a time. Figure 3 reports the IRFs to the many "cousin" shocks of the PD shock. Notably, the empirical IRFs of all variables to each of these "cousin" shocks are indistinguishable from each other and also from those to the PD shock. These "cousin" shocks are interchangeable with the PD shock in the pattern of dynamic co-movements they generate. All of them generate a strong positive co-movements across real quantities without commensurate movements in either TFP or inflation. Additionally, similar to Angeletos, Collard, and Dellas (2020), the property of interchangeability can be extended to the conditional time series produced by the PD shock and its many "cousin" shocks. Table 2 reports the correlations between the time series of various variables of interest generated by the PD shock and those generated by its many other "cousin" shocks. The very high correlations of the conditional time series generated by the PD shock and its many cousin shocks provide further evidence of the interchangeability. Finally, a complementary picture of interchangeability shows up when looking at the variance contributions of these shocks to all macroeconomic variables (Table C.1 of Online Appendix C).

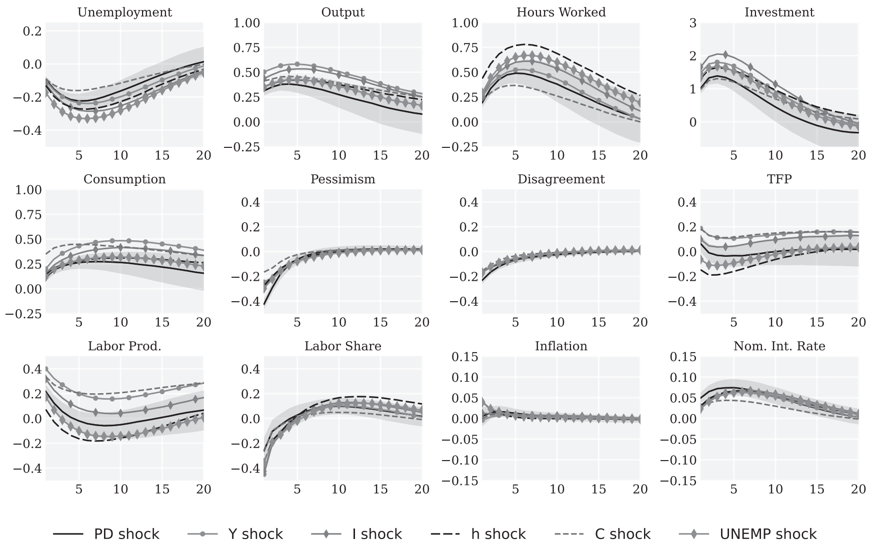

FIGURE 3. Impulse response functions to the PD shock and its many "cousin" shocks. This figure plots the impulse response functions of all variables to the identified PD shock and its many "cousin" shocks. Horizontal axis: time horizon in quarters. Shaded area: 68% HPDI of the PD shock.

TABLE 2. Correlations of conditional time series.

	Y shock	I shock	C shock	h shock	UNEMP shock
Output	0.989	0.990	0.986	0.994	0.992
Investment	0.975	0.974	0.954	0.981	0.975
Consumption	0.991	0.991	0.945	0.992	0.992
Hours worked	0.948	0.964	0.907	0.953	0.974
Unemployment	0.959	0.967	0.913	0.975	0.974
Pessimism	0.769	0.838	0.708	0.920	0.902
Disagreement	0.920	0.939	0.922	0.940	0.954

Note: Each row reports the correlation between each bandpass-filtered variable as predicted by the PD shock and that as predicted by its other "cousin" shocks.

A Summary. The empirical properties of the identified PD shock together with the interchangeability among the PD shock and its many "cousin" shocks, to quote Angeletos, Collard, and Dellas (2020), "form a rich set of cross-variable, static, and dynamic restrictions", which constitutes an "anatomy" of the business cycles extended with characteristics of households subjective beliefs, that is, pessimism and disagreement.

So far, we interpret the anatomy or specifically the identified PD shock as a structural shock of the business cycles. Hence, our empirical results suggest that the data admits a dominant shock that drives the bulk of the business-cycle fluctuations not only for real quantities but also for pessimism and disagreement. The dominant shock is able to generate positive co-movements across real quantities and counter-cyclical variations in pessimism and disagreement. Importantly, it is disconnected from either TFP or inflation at all frequencies. In other words, macroeconomic data extended with survey data on households' expectations suggest the existence of a non-inflationary aggregate demand shock that drives the bulk of the business cycles in pessimism, disagreement, and real quantities.

3.3. Discussion

In principle, the anatomy can also be interpreted as a common propagation mechanism of multiple structural shocks. Ilut and Saijo (2021) provides a theory of endogenous ambiguity that potentially supports the common-propagation-multiple-shock interpretation. Specifically, in Ilut and Saijo (2021), firms possess (Knightian) uncertainty about their productivity. Agents learn about their productivity through production in a way such that recessions are periods of less learning, which transforms into a broader range of possible models of productivity. In response, ambiguity averse agents are endogenously more pessimistic about the future outlook of the economy, which induces counter-cyclical correlated wedges. Then the economy functions as if there was a contractionary confidence shock, which makes the economy further plummet even under the flexible price model. Ambiguity and ambiguity aversion act as the internal propagation mechanism of any aggregate structural shock. And the additional propagation features positive co-movement across real quantities and counter-cyclical pessimism potentially without relying on NKPC as the propagation mechanism.[11] Therefore, the anatomy identified or specifically the PD shock in our empirical analysis can potentially be the incarnation of a linear combination of multiple aggregate shocks propagating into pessimism and real quantities through the channel of endogenous uncertainty (or ambiguity). The set of aggregate shocks can include aggregate productivity shock, monetary policy shock, financial shocks, or risk shocks under the condition that the propagation through endogenous uncertainty (or ambiguity) dominates other propagation mechanisms of the structure shock. The disconnection from productivity can be the result of a relatively low weight on aggregate TFP shock. Furthermore, the disconnection from inflation can be consistent with a relatively balanced weights on supply and demand shocks.[12]

However, what drives the business-cycle fluctuations in disagreement, which refers to the cross-sectional dispersion of beliefs of DMs inside the model, remains unaddressed in the above common-propagation-multiple-shock interpretation through the lens of Ilut and Saijo (2021). In Ilut and Saijo (2021), there are two notions of cross-sectional dispersion of beliefs: (1) one of the professional forecasters' forecasts

11. In the quantitative analysis of Ilut and Saijo (2021), nominal rigidities are still needed to generate co-movement. However, the endogenous uncertainty (or ambiguity) mechanism can easily be embedded into a model with coordination friction as in Angeletos and La'O (2013) to ensure co-movements across real quantities without relying on NKPC as the propagation mechanism.

12. Online Section IV.B of Angeletos, Collard, and Dellas (2020) provides a pedagogical example of this scenario.

on GDP growth and inflation and (2) one of the analysts' forecasts on firm profitability. Neither professional forecasters nor the analysts are DMs within the model. On the same methodological ground as Ilut and Schneider (2014), these two notions of cross-sectional dispersion of beliefs are used as the observable counterparts of ambiguity at the aggregate and firm level. Hence, there is no explanation in Ilut and Saijo (2021) for the business-cycle fluctuations of disagreement across households, who are DMs within the model. But it doesn't mean the framework of Ilut and Saijo (2021) is inconsistent with our empirical results. Extending their framework with dispersed information creates a notion of disagreement as in our paper and, at the same time, strengthens the co-movements across real quantities. To what extent, the extended framework of Ilut and Saijo (2021) can generate counter-cyclical disagreement is then a quantitative question since counter-cyclical learning does not imply counter-cyclical disagreement unambiguously.[13] Moreover, the interchangeability provides an additional test of the theory or information about the development of the model. As suggested by the above discussion, the success of Ilut and Saijo (2021) in capturing the anatomy identified in our paper requires the propagation of endogenous uncertainty (or ambiguity) to dominate standard propagation mechanisms of the multiple structural shocks. Whether or not the dominance is true depends on the nature of the structural shocks as well as what types of bells-and-whistles have been included in the model. A serious quantitative framework is needed to verify the dominance of the endogenous uncertainty in propagating structural shocks.

To summarize, in our paper, we interpret the anatomy or the identified PD shock as a structural shock, which consistently maps to ambiguity shock in our theoretical framework (Sections 4 and 5). It mainly reflects our preference in explaining the bulk of the business-cycle fluctuations in pessimism, disagreement, and real quantities with a minimal number of theoretical shocks. Meanwhile, we acknowledge the potential of an alternative interpretation of the empirical results through the lens of the theoretical framework of Ilut and Saijo (2021): the identified anatomy can also be a linear combination of multiple structural shocks propagating through the common endogenous uncertainty (or ambiguity) mechanism.

In what follows, we proceed to provide our theory of ambiguity-driven business cycles (Sections 4 and 5). We then demonstrate the quantitative success of our theory through the lens of an estimated DSGE model (Section 6) in capturing the business-cycle fluctuations in pessimism, disagreement, and real quantities.

4. Ambiguity-Driven Business Cycles: The Static Model without Capital

In this section, we construct a static general equilibrium model in the vein of Angeletos and La'O (2009), which embeds three additional key features in an otherwise standard

13. Less learning during recession, on the one hand, makes information more dispersed. On the other hand, it also reduces the responsiveness of actions to the private information. The former contributes to more disagreement, while the latter reduces disagreement. Whether or not disagreement increases in response to a reduction of learning depends on which one of the two forces dominate. Or put it differently, it depends on the signal-to-noise ratio and its business-cycle fluctuations, which cannot be determined outside a serious quantitative analysis.

RBC framework: (a) aggregate demand externalities, (b) incomplete information about the ambiguous aggregate state of the economy, and finally (c) the smooth model of ambiguity together with the ambiguity shock. We first describe the setup of the model and close up this section with a couple of remarks and interpretations of the setup.

4.1. Physical Environment, Shocks, and Information Structure

Geography, Markets, and Timing. The economy consists of a continuum of islands, indexed by $j \in \mathbb{J} = [0, 1]$ and a mainland. On each island j, there exists a continuum of firms, indexed by $(h, j) \in \mathbb{H} \times \mathbb{J} = [0, 1] \times [0, 1]$ and a continuum of workers, indexed by $(h, j) \in \mathbb{H} \times \mathbb{J} = [0, 1] \times [0, 1]$. Island firms and workers interact with each other in the locally competitive labor market for the production of differentiated island commodities indexed by j. These commodities are traded in a centralized market, later on, operated on the mainland, inhabited by a continuum of consumers, indexed by $h \in \mathbb{H} = [0, 1]$ and a large number of competitive final good producers inhabit. We assume that consumer h and a continuum of workers $\{(h, j); j \in \mathbb{J}\}$ constitute a large household indexed by $h \in \mathbb{H}$, who owns a continuum of firms $\{(h, j); j \in \mathbb{J}\}$. Thus, we ensure the existence of a representative household on the mainland and a continuum of representative firms and workers on every island.

There is only one period, say period t, which is decomposed into three stages. At stage zero, period t shocks are realized. At stage 1, island-specific competitive labor markets open up. On each island, firms make labor demand decisions and workers make labor supply decisions based on incomplete information about the ambiguous aggregate state of the economy. At stage 2, on the mainland, the centralized commodities market opens up. All uncertainty, either risk or ambiguity, is resolved. Final good producers produce and the representative household makes consumption decisions upon receiving all transfers from workers and firms on basis of perfect information. In what follows, we abstract from sub-index h without loss of generality.

Households. The representative household derives utility from the consumption of final goods, with utility function given by

$$U(C_t) = \log(C_t).$$

The consumption of final goods is financed by transfers from island workers and firms. The corresponding budget constraint of the household is such that

$$P_t C_t = \int_{\mathbb{J}} W_{j,t} N_{j,t} dj + \int_{\mathbb{J}} \Pi_{j,t} dj,$$

where P_t denotes the price of final goods, $\int_{\mathbb{J}} W_{j,t} N_{j,t} dj$ denotes total labor income and finally $\int_{\mathbb{J}} \Pi_{j,t} dj$ denotes the total realized firm profits.

Island Workers. Island j workers supply labor and receive labor income in the locally competitive labor market. Their labor income is transferred to the representative household to finance the purchase of final goods for consumption. Therefore, the utility value of labor income is given by $(U'(C_t)/P_t)W_{j,t}N_{j,t}$ where P_t is the price of final goods normalized to 1. Workers suffer disutility from the labor supply. In the absence of any uncertainty concern, island j workers' objective is, therefore, given by:

$$\frac{U'(C_t)}{P_t}W_{j,t}N_{j,t} - \frac{N_{j,t}^{1+\varepsilon}}{1+\varepsilon},$$

where ε is the inverse Frisch elasticity of labor supply.

Island Firms. Island j firms use labor only for the production of the island j commodity. The production function is given by

$$Y_{j,t} = A_{j,t}N_{j,t}^{1-\alpha},$$

where $A_{j,t}$ is the island-specific productivity and the realized profit is given by

$$\Pi_{j,t} = P_{j,t}Y_{j,t} - W_{j,t}N_{j,t}.$$

Here $W_{j,t}$ denotes the nominal wage on island j in period t and $P_{j,t}$ denotes the market price of the island j commodity to be determined at stage 2 when the centralized markets open up. Since it is assumed that it is the representative household that owns the firm, any realized profits are to be transferred back to the household for the purchase of final goods for consumption. Therefore, in the absence of any uncertainty concerns, island j firms care about the consumer valuation of their profits given by

$$\frac{u'(C_t)}{P_t}\Pi_{j,t}.$$

Final-Good Producers. The competitive final-good sector employs a constant elasticity of substitution (CES) production technology:

$$Y_t = \left(\int_{\mathbb{J}} Y_{j,t}^{\frac{\theta-1}{\theta}}\,dj\right)^{\frac{\theta}{\theta-1}},$$

where θ is the elasticity of substitution among island commodities. It also controls the strength of aggregate demand externalities. Therefore, the demand function for the island j commodity is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t,$$

where $P_t \equiv (\int_{\mathbb{J}} P_{j,t}^{1-\theta}\,dj)^{\frac{1}{1-\theta}}$ denotes the price of final goods that is normalized to 1.

Productivity and Ambiguity Shocks. Aggregate productivity $a_t \equiv \log A_t$ follows a normal distribution with mean ω_t and variance σ_ξ^2

$$a_t \sim \mathcal{N}(\omega_t, \sigma_\xi^2).$$

Objectively, the mean of the aggregate productivity shock is zero, that is, $\omega_t = 0$. However, DMs cannot fully "understand" it. Instead, they are ambiguous about it. Specifically, they believe that anything along the real-line can be a potential candidate for ω_t. And they possess a common mean-zero[14] normal prior belief about $\omega_t \in \mathbb{R}$:

$$\omega_t \sim \mathcal{N}\left(0, e^{\psi_t}\right),$$

where ψ_t measures the amount of ambiguity perceived by the DMs[15] such that

$$\psi_t = \overline{\psi} + \tau_t \quad \text{with} \quad \tau_t \sim \mathcal{N}\left(0, \sigma_\tau^2\right).$$

Here $\overline{\psi}$ denotes the amount of ambiguity perceived by all DMs at the ambiguous steady state (Amb.-SS).[16] And we interpret τ_t as the ambiguity shock, which is normally distributed with mean 0 and variance σ_τ^2.

Island-specific productivity, $a_{j,t} \equiv \log A_{j,t}$, equals to the aggregate productivity plus an idiosyncratic productivity shock $\iota_{j,t}$:

$$a_{j,t} = a_t + \iota_{j,t}.$$

Idiosyncratic productivity shocks $\iota_{j,t}$ are assumed to be i.i.d normally distributed with mean 0 and variance σ_ι^2.

Information Structure. Denote $\mathcal{I}_{t,0}$, $\mathcal{I}_{j,t,1}$, and $\mathcal{I}_{t,2}$ as the information sets that are available to all DMs at stage 0 of period t, are only available to island j DMs at stage 1 of period t and are available to all DMs at stage 2 of period t, respectively. We define these information sets by

$$\mathcal{I}_{t,0} = \{\psi_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \{a_{j,t}\} \qquad \mathcal{I}_{t,2} = \cup_j \mathcal{I}_{j,t,1} \cup \{\omega_t\}. \tag{1}$$

A couple of implicit assumptions are made here. First of all, information is symmetric within each island but is asymmetric across islands. Second, the ambiguity shock τ_t occurs at the beginning of each period t and is common knowledge to all DMs. Third, at stage 1 of period t, island j productivity $a_{j,t}$ is only accessible for island j DMs. Therefore, $a_{j,t}$ serves as the private information of island j DMs about the aggregate productivity a_t, which is ambiguous. Thus, labor supply and demand decisions on each island are made under incomplete information about the ambiguous aggregate state of the economy. Fourth, $\mathcal{I}_{t,2}$ contains the complete set of local information because commodities prices would perfectly reveal the island-specific productivities.

14. The objective model ω_t is assumed to be inside the set of possible models of all DMs. Therefore, we rule out any misspecification concerns and focus on ambiguity. See Peter Hansen and Marinacci (2016) for a detailed discussion of the differences between misspecification and ambiguity.

15. Maccheroni, Marinacci, and Ruffino (2013) propose to use the variance of the ex-ante expected utility of a particular model to quantify the amount of ambiguity in the general information structure, which is shown to be consistent with a quadratic approximation akin to the Arrow–Pratt approximation. Our measure of the amount of ambiguity is consistent with theirs ordinally under the normality assumption.

16. Ambiguous steady state refers to the state into which the economy converges in the absence of any shocks but taking into account the existence of ambiguity.

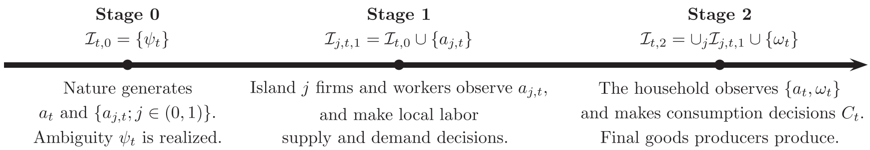

FIGURE 4. Timeline for period t of the static model without capital.

Fifth, all uncertainty, either risk (state uncertainty about a_t) or ambiguity (model uncertainty about ω_t) is fully resolved at stage 2 of period t. Hence, consumption decisions are made under perfect information.[17] Finally, for simplicity and tractability, we consider an environment with private information only. In the Online Appendix G, we demonstrate that the model can be easily extended with public information about aggregate productivity without changing the qualitative predictions of the model.

We close the current sub-section by the timeline of our model in Figure 4.

4.2. *Preferences and Interim Belief System*

The preference of the representative household is represented by the smooth model of ambiguity (Klibanoff, Marinacci, and Mukerji 2005).[18] In addition, with ambiguity averse preferences, Bayesian updating leads to dynamic inconsistency. To restore the dynamic consistency across stages, we employ the smooth rule of updating proposed by Hanany and Klibanoff (2009). Recall that at stage 2, when all uncertainty has been resolved, the model collapses into the standard perfect information business-cycle model. In what follows, we formulate the relevant workers' and firms' problems upon carefully describing the preferences and beliefs of all DMs at stage 1 of period t.

Worker Problem at Stage 1. At stage 1 of period t, workers on island j solve the following problem:

$$\max_{N_{j,t}} \int_{\mathbb{R}} \varphi\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t} W_{j,t} N_{j,t} - \frac{N_{j,t}^{1+\varepsilon}}{1+\varepsilon}\right]\right) \tilde{f}^{w}_{j,t,1}(\omega_t) d\omega_t, \tag{2}$$

where $N_{j,t}$ denotes the labor supply of island j workers and $W_{j,t}$ denotes the nominal wage in the island-specific competitive labor market.

Here, $\varphi(x)$ is a strictly increasing and concave function, whose curvature captures DMs' taste about ambiguity, namely the degree of ambiguity aversion. In addition, $\mathbb{E}^{\omega_t}_{j,t,1}[\cdot]$ denotes the mathematical expectation conditioned on $\mathcal{I}_{t,1}$ under a particular

17. The assumption that all period t uncertainty is resolved at the second stage of period t is to ensure tractability. However, most of the key messages delivered in this paper do not rely on this particular assumption about information structure.

18. The smooth model features a two-way separation between the amount of ambiguity (the characteristics of subjective belief) and the degree of ambiguity aversion (the characteristics of DMs' tastes).

model ω_t for the mean of the aggregate productivity shock. Finally, $\tilde{f}^{w}_{j,t,1}(\omega_t)$ stands for the posterior belief about possible models ω_t that follows the extended smooth rule of updating in the spirit of Hanany and Klibanoff (2009):

$$\tilde{f}^{w}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}[U(C_t)]\right)}{\varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t} W_{j,t} N_{j,t} - \frac{N^{1+\varepsilon}_{j,t}}{1+\varepsilon}\right]\right)}}_{\text{Weights}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t(\omega_t)}_{\text{Bayesian Kernel}}. \tag{3}$$

Here $f(a_{j,t}|\omega_t)$ is the conditional probability density function of $a_{j,t}$ under a particular model ω_t, which is the normal density with mean ω_t and variance $\sigma^2_\xi + \sigma^2_\iota$, and $f_t(\omega_t)$ stands for the period t prior belief about ω_t, which is the normal density with mean 0 and variance e^{ψ_t}. Relative to the standard Bayesian updating, the extended smooth rule puts more weight on the model that provides higher marginal incentive to act ex-ante (stage 0) from the perspective of the representative consumer as compared to its ex-post (stage 2) counterparts from the workers' perspective:

$$\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}\left[U(C_t)\right]\right) > \varphi'\left(\mathbb{E}^{\omega_t}_{t,2}\left[\frac{U'(C_t)}{P_t} W_{j,t} N_{j,t} - \frac{N^{1+\varepsilon}_{j,t}}{1+\varepsilon}\right]\right).$$

Such a re-weighting process in posterior belief, on the one hand, aligns the incentives to act ex-ante and ex-post, which ensures dynamic consistency across stages within a period. On the other hand, it ensures that workers share the same belief system with the representative consumer, which ensures the tractability of the model without loss of any generality.

Firm Problem At Stage 1. Island j firms decide how much labor to hire by solving the following firm problem:

$$\max_{N_{i,j,t}} \int_{\mathbb{R}} \varphi\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t}\left(P_{j,t} Y_{j,t} - W_{j,t} N_{j,t}\right)\right]\right) \tilde{f}^{f}_{j,t,1}(\omega_t) d\omega_t. \tag{4}$$

Note that there are a continuum of firms on island j. Therefore, island j firms take $P_{j,t}$ as given when making labor demand decisions. Furthermore, the posterior belief about possible models ω_t of island j firms follows an extended smooth rule of updating given by

$$\tilde{f}^{f}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}[U(C_t)]\right)}{\varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t}\left(P_{j,t} Y_{j,t} - W_{j,t} N_{j,t}\right)\right]\right)}}_{\text{Weights}} \underbrace{f(a_{j,t}|\omega_t) f_t(\omega_t)}_{\text{Bayesian Kernel}}. \tag{5}$$

Under our extended smooth rule of updating, firms' incentives to act align with those of the representative consumer ex-ante. Thus, we can ensure dynamic consistency from the perspective of the representative consumer. Namely, if we allowed the representative consumer to make ex-ante contingency production plans for island firms, they would be respected ex-post by the firms. [19]

Finally, we assume that $\varphi(x)$ takes the constant absolute ambiguity aversion (CAAA) form for simplicity and tractability:

ASSUMPTION 1 (CAAA). We assume $\varphi(x) = -\frac{1}{\lambda}\exp(-\lambda x)$ where $\lambda \geq 0$ measures the degree of ambiguity aversion of all DMs.

4.3. *Remarks and Interpretations*

We conclude this section with two remarks and interpretations of the three key features of our model.

(1) We assume no ambiguity about local economic conditions in the sense that firms and workers on island j have perfect information about the productivity of island j, while they have incomplete information about the aggregate productivity, which is ambiguous. Therefore, if local economic decisions are made solely depending on the expectations about local economic conditions, output or labor will not respond to the ambiguity shocks at all. This is the reason why we need aggregate demand externalities in the general equilibrium model.

(2) In our model, incomplete information about the ambiguous aggregate productivity a_t can be translated into incomplete information about the ambiguous aggregate demand within a general equilibrium environment that features aggregate demand externalities. Then, fluctuations in the amount of ambiguity, namely ambiguity shock τ_t, generate fluctuations in island j DMs' subjective beliefs about aggregate demand conditions, which eventually map into fluctuations in aggregate quantities. In this sense, we can formally interpret ambiguity shock as aggregate demand shock. Importantly, we set up our model within the RBC framework, which can be regarded as the flexible price benchmark of the New Keynesian (NK) model with Calvo pricing. As will be evident in the Sections 5 and 6, unlike

19. Note that we formulate the firms' problem in such a way that the firms are ambiguity averse by themselves. We can justify the above formulation of the firm problem by arguing that firms are maximizing the shareholder value and use the stochastic discount factor to evaluate cash flows in different times and states. Therefore, they behave as if they are ambiguity averse by themselves and share the same belief with their shareholders when evaluating the marginal benefit of labor demand. The additional concavity introduced by the φ function manifests the former point, and the extended smooth rule of updating takes care of the latter. Therefore, we can alternatively formulate the firms' problem by

$$\max_{N_{i,j,t}} \int_{\mathbb{R}} \mathbb{E}_{j,t}^{\omega_t}[\mathrm{SDF}_t(P_{j,t}Y_{i,j,t} - W_{j,t}N_{i,j,t})] f_{j,t,1}(\omega_t) d\omega_t, \tag{6}$$

where the stochastic discount factor SDF_t is given by

$$\mathrm{SDF}_t \equiv \varphi'\left(\mathbb{E}_{t,0}^{\omega_t}[U(C_t)]\right)\frac{U'(C_t)}{P_t}.$$

Here, the stochastic discount factor does not only take care of the risk attitude $U'(C_t)/P_t$ but also the ambiguity attitude $\varphi'(\mathbb{E}_{t,0}^{\omega_t}[U(C_t)])$. The two formulations (4) and (6) are isomorphic to each other.

aggregate demand shocks in the NK framework, ambiguity shock can generate co-movement across real quantities even under flexible price allocations. In other words, ambiguity shock is a non-inflationary aggregate demand shock, which generates co-movements across real quantities without relying on the NKPC as the propagation mechanism. It is then straight-forward to anticipate that the impacts of ambiguity shock would remain nearly unchanged in a model with nominal rigidities, such as Calvo pricing.

5. Impacts of the Ambiguity Shock

In this section, we characterize the equilibrium of our model with a set of optimality conditions that jointly describe the equilibrium allocations and beliefs of all relevant DMs.[20] We then demonstrate how to solve the equilibrium allocation associated with these optimality conditions. Finally, we discuss the impacts of the ambiguity shock in generating the empirically relevant co-movements among pessimism, disagreement, and real quantities.

5.1. Equilibrium Characterization

We can characterize the equilibrium with a set of optimality conditions. First of all, within the island j labor market, the optimal labor supply is governed by the following condition

$$\chi N_{j,t}^{\varepsilon} = W_{j,t} \int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t}[u'(C_t)] \tilde{f}_{j,t,1}(\omega_t) d\omega_t. \tag{7}$$

Workers on island j equate the subjective valuation of the marginal benefit of labor with marginal the disutility of labor at stage 1. On the other side of the labor market, the optimal labor demand condition is given by

$$\begin{aligned} W_{j,t} &\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t}[u'(C_t)] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \\ &= (1-\alpha) \frac{Y_{j,t}}{N_{j,t}} \left(\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t}[u'(C_t) P_{j,t}] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \right). \end{aligned} \tag{8}$$

Firms on island j equate the subjective valuation of the marginal cost of labor with the marginal benefit at stage 1. Unlike expected utility preferences, ambiguity aversion implies that when evaluating marginal effects at stage 1 of period t, firms and workers on island j employ a distorted posterior belief about the possible models:

$$\tilde{f}_{j,t,1}(\omega_t) \propto \underbrace{\varphi'\left(\mathbb{E}_{t,0}^{\omega_t}[U(C_t)]\right)}_{\text{Belief Distortion}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t(\omega_t)}_{\text{Bayesian Kernel}}. \tag{9}$$

20. The market equilibrium is defined by Definition E.1 in Online Appendix E.

It says that whenever a model ω_t generates a lower ex-ante (stage 0) expected utility for the representative household, local DMs tend to regard it as the more likely one in their distorted posteriors. Put differently, ambiguity aversion implies a pessimistic belief about the possible models when DMs are making decisions.[21]

Combing (7) and (8), the labor market equilibrium can be summarized by the following key equation for labor:[22]

$$\chi N_{j,t}^{\varepsilon} = \left(\underbrace{\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[u'(C_t) \left(\frac{Y_{j,t}}{Y_t} \right)^{-\frac{1}{\theta}} \right] \tilde{f}_{j,t,1}(\omega_t) d\omega_t}_{\text{marginal utility of island } j \text{ commodity}} \right) \left(\underbrace{(1-\alpha) \frac{Y_{j,t}}{N_{j,t}}}_{\text{marginal productivity}} \right). \tag{10}$$

The left hand side (LHS) is the marginal disutility of labor and the right hand side (RHS) is the multiplication of (a) the marginal utility of the island j commodity and (b) the marginal productivity of island labor. The key equation says, in equilibrium, that the subjective valuation of the private benefit of labor equates the private cost of labor at stage 1. A similar condition also shows up in Angeletos and La'O (2009) and Angeletos, Iovino, and La'O (2016). There are two main differences between ours and theirs. First of all, there is one additional integration of different models due to the existence of ambiguity. Second, DMs use a distorted posterior belief due to ambiguity aversion. As will become evident later, these two differences together allow us to build a bridge between variations in ambiguity to confidence, which is the core mechanism of the model.

Using the island production function $Y_{j,t} = A_{j,t} N_{j,t}^{1-\alpha}$ and the market clearing condition for final goods $Y_t = C_t$, we can transform (10) into a fixed point condition for allocation $\{Y_{j,t}\}_{j \in J}$:

$$\chi Y_{j,t}^{\frac{1+\varepsilon}{1-\alpha}-1+\frac{1}{\theta}} = (1-\alpha) A_{j,t}^{\frac{1+\varepsilon}{1-\alpha}} \left(\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[Y_t^{\frac{1}{\theta}-1} \right] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \right), \tag{11}$$

where the distorted posterior belief $\tilde{f}_{j,t,1}(\omega_t)$ is given by

$$\tilde{f}_{j,t,1}(\omega_t) \propto \underbrace{\varphi'\left(\mathbb{E}_{t,0}^{\omega_t}[U(Y_t)]\right)}_{\text{Belief Distortion}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t(\omega_t)}_{\text{Bayesian Kernel}}. \tag{12}$$

To ensure the strategic complementarity in productions across islands, we make the following parametric restriction for the static model without capital:[23]

21. Note that the firms on island j have the same distorted posterior belief over the set of possible models as island j workers, which is due to the extended smooth rule.

22. A detailed derivation can be found in Appendix A.

23. To see why this is the case, observe that under perfect information, (11) can be simplified into

$$\chi Y_{j,t}^{\frac{1+\varepsilon}{1-\alpha}-1+\frac{1}{\theta}} = \left(\frac{\eta-1}{\eta}\right)(1-\alpha) A_{j,t}^{\frac{1+\varepsilon}{1-\alpha}} Y_t^{\frac{1}{\theta}-1}.$$

It is straightforward to show $\partial Y_{j,t}/\partial Y_t > 0$ if and only if $1/\theta - 1 > 0$.

ASSUMPTION 2 (*Strategic Complementarity*). It is assumed that $1/\theta > 1$ when there is no capital.

An increase in the output of all other islands $k \neq j \in \mathbb{J}$, on the one hand, raises the demand for island j commodities because households have more labor income from all other islands. This is the so-called aggregate demand externality. However, on the other hand, it also generates upward pressure on the wage rate of island j due to the wealth effect of labor supply. Assumption 2 ensures that the wealth effect of labor supply is so weak that the aggregate demand externality dominates in equilibrium.

DEFINITION 1 (*Conditional Log-Normal Equilibrium*). An allocation $\{\{Y_{j,t}\}_{j\in\mathbb{J}}, Y_t\}$ constitutes a conditional log-normal equilibrium if $Y_{j,t}|\psi_t$ and $Y_t|\psi_t$ are log-normally distributed.

The technical complication here is that in equilibrium, the distorted posterior belief $\tilde{f}_{j,t,1}(\omega_t)$ is not orthogonal to allocations. They have to be solved simultaneously in equilibrium. As a result, the equilibrium of the economy is the solution to the double fixed point conditions: One condition solves (11) characterizing the equilibrium cross-sectional allocation $\{Y_{j,t}\}_{j\in\mathbb{J}}$ conditional on any distorted posterior belief about possible models $\tilde{f}_{j,t,1}(\omega_t)$ and the other solves (12) characterizing an equilibrium distorted posterior belief about the possible models conditional on any allocation $\{Y_{j,t}\}_{j\in\mathbb{J}}$ of the economy.

The complication can be resolved by focusing on a particular type of equilibrium—the conditional log-normal equilibrium as defined in Definition 1. On the one hand, the conditional log-normal equilibrium embeds the standard log-normal equilibrium or log-linearized equilibrium as a special case when there are no ambiguity shocks while, on the other hand, it can be justified, in a self-fulfilling fashion, by Lemma A.1 and A.2 in Appendix A. The following proposition characterizes the conditional log-normal equilibrium.

PROPOSITION 1 (Equilibrium Characterization). *There exists a unique symmetric conditional log-normal equilibrium where the allocation $\{Y_{j,t}, Y_t\}_{j\in\mathbb{J}}$ is such that*

$$y_{j,t} \equiv \ln Y_{j,t} = \left(\underbrace{y^* + \bar{h}_y(\bar{\psi},\lambda)}_{\text{Ambiguous SS}}\right) + \underbrace{\kappa_{ya_j}(\psi_t)}_{\text{Use of Private Info.}}\ a_{j,t} + \underbrace{\hat{h}_y(\psi_t,\lambda)}_{\text{Impact of Amb. Shock}}$$

and

$$y_t \equiv \ln Y_t = \left(\underbrace{y^* + \bar{h}_y(\bar{\psi},\lambda)}_{\text{Ambiguous SS}}\right) + \underbrace{\kappa_{ya_j}(\psi_t)}_{\text{Use of Private Info.}}\ a_t + \underbrace{\hat{h}_y(\psi_t,\lambda)}_{\text{Impact of Amb. Shock}},$$

where $y^* + \bar{h}_y(\overline{\psi}, \lambda)$ *denotes the output level at the ambiguous steady state;* $\kappa_{ya_j}(\psi_t)$ *refers to the use of private information, which is a function of the amount of ambiguity* ψ_t*; and* $\hat{h}_y(\psi_t; \lambda)$ *denotes the impact of the ambiguity shock on output, which is a function of the amount of ambiguity* ψ_t *and the degree of ambiguity aversion* λ*, satisfying*

$$\hat{h}_y(\overline{\psi}, \lambda) = 0.$$

Finally, the distorted posterior belief about the possible model ω_t *is a normal distribution with mean* $\mu_{j,t}$ *and variance* σ_t^2 *such that*

$$\mu_{j,t} = \left(\frac{e^{\psi_t}}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) a_{j,t} + \left(\frac{\sigma_\xi^2 + \sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) g_\mu(\psi_t, \lambda)$$

and

$$\sigma_t^2 = \left(\frac{\sigma_\xi^2 + \sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) e^{\psi_t},$$

where the distortion in mean $g_\mu(\psi_t, \lambda)$ *is given by*

$$g_\mu(\psi_t, \lambda) = -\lambda \kappa_{ya_j}(\psi_t) e^{\psi_t}. \tag{13}$$

Proof. See Appendix A. □

5.2. *The Dual Impacts of Ambiguity Shock*

To build up the economic intuitions behind impacts of ambiguity shocks, we demonstrate a game-theoretic interpretation of the equilibrium of our business-cycle model, which resembles the beauty contest in Morris and Shin (2002) and Angeletos and Pavan (2007), but with a distorted information structure to capture the belief distortions due to ambiguity aversion.

PROPOSITION 2. *The equilibrium allocation* $\{Y_{j,t}, Y_t\}_{j \in \mathbb{J}}$ *is identical to that of a beauty contest such that*

$$y_{j,t} = \kappa_a a_{j,t} + \kappa_y \widetilde{\mathbb{E}}_{j,t}[y_t],$$

where the coefficients κ_a *and* κ_y *are such that*

$$\kappa_a = \frac{\frac{1+\varepsilon}{1-\alpha}}{\frac{1+\varepsilon}{1-\alpha} - 1 + \frac{1}{\theta}} \qquad \kappa_y = \frac{\frac{1}{\theta} - 1}{\frac{1+\varepsilon}{1-\alpha} - 1 + \frac{1}{\theta}} \in (0, 1).$$

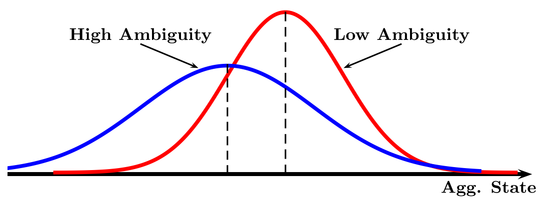

FIGURE 5. Dual impacts of ambiguity shock: Main mechanism.

The information structure is distorted such that

$$\tilde{a}_{j,t} = \tilde{a}_t + \tilde{\iota}_{j,t}, \quad \tilde{\iota}_{j,t} \sim \mathcal{N}(0, \sigma_\iota^2)$$
$$\tilde{a}_t \sim \mathcal{N}\left(g_\mu(\psi_t, \lambda), \sigma_\xi^2 + e^{\psi_t}\right),$$

where distortion $g_\mu(\psi_t, \lambda)$ are given by (13) and satisfies the following

$$g_\mu(\psi_t, \lambda) \leq 0, \quad g_\mu(-\infty, \lambda) = 0, \quad g_\mu(\psi_t, 0) = 0, \quad \frac{\partial g_\mu(\psi_t, \lambda)}{\partial \psi_t} < 0.$$

Proof. See Appendix A. □

As in the beauty contest, island output $y_{j,t}$ is the linear combination of island productivity $a_{j,t}$ and the island j expectation of aggregate output. The former controls the marginal cost of production on island j and the latter manifests island j's forecast about its demand conditions. Here, κ_y corresponds to the notion of the coordination motive in the beauty contest literature. Its magnitude $\kappa_y \in (0, 1)$ ensures complementarity in action and uniqueness in allocations, once we fix a distorted information structure.

However, unlike the standard beauty contest, the perceived distribution of aggregate productivity is distorted both in mean and variance due to ambiguity aversion. Figure 5 plots the perceived as if distributions[24] of the aggregate fundamental for a low level of ambiguity, that is, ψ_t is small, and for a high level of ambiguity, that is, ψ_t is large. It can be shown that with an adverse ambiguity shock, namely an increase in ψ_t, DMs become more uncertain about the aggregate fundamental. At the same time, it makes DMs more pessimistic about the aggregate fundamental. We call these the dual impacts of ambiguity shocks.

24. These distributions are called "as if" because they are the subjective beliefs about aggregate fundamentals that would deliver the same allocations as our baseline model when DMs have expected utility preferences.

5.3. *Pessimism, Disagreement, and Real Quantities*

With aggregate demand externality, a distorted prior belief about aggregate productivity $\tilde{a}_t$ translates into a distorted prior belief about aggregate demand. Therefore, an adverse ambiguity shock makes all DMs believe that the aggregate demand drops and is more volatile in their priors. The former maps into lower output, either of a particular island or at the aggregate, while the latter maps into an increased incentive in the use of private information when making the expectation about aggregate demand, hence when making labor demand and supply decisions. We summarize these results in the following proposition

PROPOSITION 3. *If DMs are ambiguity averse, that is, $\lambda > 0$, an adverse ambiguity shock that increases the amount of ambiguity ψ_t generates lower aggregate output in the sense that*

$$\frac{\partial \hat{h}_y(\psi_t, \lambda)}{\partial \psi_t} < 0. \tag{14}$$

Moreover, the equilibrium use of private information $\kappa_{ya_j}(\psi_t)$ is an increasing function of the amount of ambiguity ψ_t:

$$\frac{\partial \kappa_{ya_j}(\psi_t)}{\partial \psi_t} > 0.$$

Proof. See Appendix A. □

At the core of understanding (14) is the increased degree of pessimism about aggregate demand. There are two forces at work, one fundamental and one strategic. An adverse ambiguity shock, on the one hand, increases the amount of ambiguity perceived by all DMs. In response, DMs behave as if they have more pessimistic concerns about aggregate productivity. This is the fundamental or direct channel. On the other hand, an adverse ambiguity shock induces DMs on the other islands to use more of their private information when making output decisions, which makes aggregate demand respond more to the ambiguous aggregate productivity. Under aggregate demand externality, it further increases the degree of the pessimism of local DMs. This is the strategic or indirect channel. An adverse ambiguity shock raises all DMs' degrees of pessimism through the fundamental and strategic channels, which eventually drives down the output. Finally, since $y_{j,t} = a_{j,t} + (1-\alpha)n_{j,t}$, it must be the case that hours worked also drops:

$$\frac{\partial \hat{h}_n(\psi_t, \lambda)}{\partial \psi_t} < 0.$$

Define the output forecasts of island j DMs and their noisy rational expectation counterparts as follows:

$$\widetilde{\mathbb{E}}_{j,t}[y_t] \equiv \int_{\mathbb{R}} E^{\omega_t}_{j,t}[y_t] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \qquad \mathbb{E}_{j,t}[y_t] \equiv \int_{\mathbb{R}} E^{\omega_t}_{j,t}[y_t] f_{j,t,1}(\omega_t) d\omega_t.$$

The local output forecasts $\widetilde{\mathbb{E}}_{j,t}[y_t]$ are defined on the basis of the distorted posterior belief about possible models $\tilde{f}_{j,t,1}(\omega_t)$, while the noisy rational expectation counterparts $\mathbb{E}_{j,t}[y_t]$ are based on the Bayesian posteriors $f_{j,t,1}(\omega_t)$. By doing so, we implicitly assume that ambiguity averse DMs would use their subjective pessimistic beliefs to make forecasts. We then define pessimism (P) as the gap in cross-sectional mean between local forecasts and their noisy rational expectation counterparts:

$$P(\psi_t, \lambda) \equiv \int_{\mathbb{J}} \mathbb{E}_{j,t}[y_t]dj - \int_{\mathbb{J}} \widetilde{\mathbb{E}}_{j,t}[y_t]dj,$$

and define disagreement (D) as the cross-sectional dispersion of output forecasts across islands:

$$\mathrm{D}(\psi_t) \equiv \int_{\mathbb{J}} \left(\widetilde{\mathbb{E}}_{j,t}[y_t] - \int_{\mathbb{J}} \widetilde{\mathbb{E}}_{j,t}[y_t]dj \right)^2 dj.$$

We can express pessimism (P) as follows:

$$\mathrm{P}(\psi_t, \lambda) = \kappa_{ya_j}(\psi_t, \lambda) \int_{\mathbb{J}} \left[\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t}[a_t] f_{j,t,1}(\omega_t) d\omega_t - \int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t}[a_t] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \right] dj,$$

where $\int_{\mathbb{J}} \int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t}[a_t] \tilde{f}_{j,t,1}(\omega_t) d\omega_t dj$ denotes the cross-sectional mean of local DMs' beliefs about aggregate productivity and $\int_{\mathbb{J}} \int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t}[a_t] f_{j,t,1}(\omega_t) d\omega_t dj$ denotes the noisy rational expectation counterparts. When there is an adverse ambiguity shock, all DMs, in their priors, become more pessimistic about the aggregate fundamental, that is, $|g_\mu(\psi_t, \lambda)| = -g_\mu(\psi_t, \lambda)$ increases. On the other hand, all DMs perceive the aggregate fundamental to become more volatile, that is, $\sigma_\xi^2 + e^{\psi_t}$ increases. The former increases the economy-wide pessimism. However, the latter reduces the use of pessimistic priors. In equilibrium, the former dominates the latter, implying that all DMs are becoming more pessimistic about aggregate productivity when making decisions.

$$\int_{\mathbb{J}} \int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t}[a_t] \left(f_{j,t,1}(\omega_t) - \tilde{f}_{j,t,1}(\omega_t) \right) d\omega_t dj = \left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + e^{\psi_t} + \sigma_\iota^2} \right) |g_\mu(\psi_t, \lambda)|.$$

Moreover, all DMs also understand that others all perceive the aggregate fundamental as being more volatile. Hence, they understand that all the others would use more of their private information when making output decisions. Therefore, they know that aggregate output will respond more to the aggregate fundamental, that is, $\kappa_{ya_j}(\psi_t, \lambda)$ increases. It raises output forecasts' reliance on the pessimistic belief about aggregate productivity, which even further increases pessimism.[25]

25. In our model, island j DMs have perfect information about their own productivity. An increase in the amount of ambiguity depresses island j DMs' belief about the aggregate productivity without any changes

At the same time, we can express disagreement (D) as follows:

$$\mathrm{D}(\psi_t) = \kappa_{ya_j}^2(\psi_t)\left(\frac{\sigma_\xi^2 + e^{\psi_t}}{\sigma_\xi^2 + e^{\psi_t} + \sigma_\iota^2}\right)^2 \sigma_\iota^2.$$

An adverse ambiguity shock makes firms and workers on all islands believe, in their subjective priors, that the aggregate fundamental is more volatile, which increases the incentive to use private information when forming expectations about aggregate demand conditions. This maps into an increased responsiveness of island output $y_{j,t}$ to island productivity $a_{j,t}$ because it is $a_{j,t}$ that serves as the private information about aggregate demand for island j DMs. Hence, aggregate output y_t responds more to aggregate productivity a_t. From the perspective of forecaster j, an increase in κ_{ya_j} implies that "there is more to be estimated". Moreover, when he estimates the aggregate productivity, he tends to rely more on his private information $a_{j,t}$ because he believes, in his as if subjective prior, that the aggregate fundamental is now more volatile. These two in combination increase the responsiveness of forecaster j's forecast to private information $a_{j,t}$, which eventually leads to a higher cross-sectional dispersion in output forecasts ex-ante, that is, an increase in disagreement.[26]

We close up this section by summarizing the impacts of the ambiguity shock in Proposition 4: an adverse ambiguity shock can generate a recession with worsened pessimism (P) and heightened disagreement (D).

PROPOSITION 4 (Impacts of Ambiguity Shocks). *If decision-makers are ambiguity averse, that is, $\lambda > 0$, an adverse ambiguity shock that increases the amount of ambiguity ψ_t generates*

(1) *higher pessimism;*
(2) *higher disagreement;*
(3) *lower aggregate output, hours worked, and consumption.*

Proof. Straightforward following Proposition 3 and the above analysis. □

6. Quantitative Analysis: The DSGE

In this section, we demonstrate the quantitative potential of our theoretical framework through the lens of a DSGE model. We focus on a flexible price setup to capture the fact

in beliefs about their local productivities. The patterns of variations in beliefs due to the ambiguity shock in our model are isomorphic to those of a negative confidence shock under the heterogenous prior setup à la Angeletos, Collard, and Dellas (2018).

26. Note that the economy itself does not become more dispersed. It is the increased responsiveness to idiosyncratic shocks that drives up the cross-sectional dispersion. This differentiates our paper from the theory of the uncertainty shock as in Bloom (2009) and Bloom et al. (2018), which take fluctuations in dispersion as model input rather than model output.

that dynamic co-movements produced by the ambiguity shock, which is the theoretical counterpart of the PD shock, are orthogonal to inflation or NKPC.[27] We also introduce a set of frictions that are otherwise standard in most DSGE models including (1) habit formation, (2) investment adjustment cost, and (3) variable capital utilization. The habit formation and the investment adjustment cost are to capture the hump-shape IRFs of real quantities to the ambiguity shock and the variable capital utilization ensures that there are mild variations in labor productivity to the ambiguity shock. The DSGE model is estimated by matching the theoretical IRFs of the ambiguity shock with their empirical counterparts, namely those of the identified PD shock.

6.1. Model Setup

The Household. The representative household consists of a consumer, a continuum of island workers indexed by $j \in \mathbb{J}$ and a continuum of island capital managers $j \in \mathbb{J}$. Workers and capital managers with index j inhabit island j and interact with island firms in locally competitive factor markets.

The representative household owns the island-specific physical capital $\bar{K}_{j,t}$. At stage 1 of period t, capital manager j chooses the capital utilization rate $u_{j,t}$, which transforms physical capital $\bar{K}_{j,t}$ into effective capital $K_{j,t}$:

$$K_{j,t} = u_{j,t} \bar{K}_{j,t}.$$

Effective capital $K_{j,t}$ is then rented out to island firms in locally competitive capital markets at price $R_{j,t}$. The real cost of capital utilization per unit of capital is given by $\Xi(u_{j,t})$. We assume that $u_{j,t} = 1$ in the Amb.-SS and the utilization-cost function $\Xi(u_{j,t})$ satisfies that $\Xi(1) = 0$ and

$$u \frac{\Xi''(u)}{\Xi'(u)} = \frac{\xi}{1-\xi},$$

where the parameter $\xi \in [0, 1]$.

The flow budget constraint of the representative household is then given by

$$\begin{aligned} C_t + \int_{\mathbb{J}} I_{j,t} dj + T_t = \int_{\mathbb{J}} W_{j,t} N_{j,t} dj + \int_{\mathbb{J}} R_{j,t} u_{j,t} \bar{K}_{j,t} dj \\ + \int_{\mathbb{J}} \int_{\mathbb{I}} \Pi_{i,j,t} di dj - \Xi(u_{j,t}) \bar{K}_t, \end{aligned} \tag{15}$$

where $\int_{\mathbb{J}} R_{j,t} u_{j,t} \bar{K}_{j,t} dj$ and $\int_{\mathbb{J}} W_{j,t} N_{j,t} dj$ denote the total capital and labor income from all islands; $\int_{\mathbb{J}} \int_{\mathbb{I}} \Pi_{i,j,t} di dj$ are the total transfers of firm profits from all islands; and T_t are the lump-sum taxes.

At stage 2 of period t, the consumer makes consumption-saving decisions, where savings are in the form of island-specific investment. Therefore, the supply of physical

27. Alternatively, the DSGE model considered in the section can be understood as the flexible price benchmark of the NK model with Calvo frictions.

capital on island j in period $t+1$ is pre-determined at stage 2 of period t. The accumulation of physical capital is subject to the investment adjustment cost following Christiano, Eichenbaum, and Evans (2005):

$$\bar{K}_{j,t+1} = (1-\delta)\bar{K}_{j,t} + I_{j,t}\left(1 - \Phi\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right), \tag{16}$$

where $\Phi(\cdot)$ satisfies $\Phi'(\cdot) > 0$, $\Phi''(\cdot) > 0$, $\Phi(1) = \Phi'(1) = 0$, and $\Phi''(1) = \varphi$. Specifically, we assume that $\Phi(x) = 0.5\varphi(x-1)^2$.

Production. For any island $j \in \mathbb{J} \equiv [0,1]$, index firms by $i \in \mathbb{I} \equiv [0,1]$. The output of firm i on island j is given by

$$Y_{i,j,t} = A_{j,t} N_{i,j,t}^{1-\alpha} K_{i,j,t}^{\alpha},$$

where $A_{j,t}$ is the island-specific productivity, $N_{i,j,t}$ is the labor demand, and $K_{i,j,t}$ is the demand for effective capital. The profit of firm i on island j is given by

$$\Pi_{i,j,t} = P_{i,j,t} Y_{i,j,t} - W_{j,t} N_{i,j,t} - R_{j,t} K_{i,j,t}, \tag{17}$$

where $W_{j,t}$ and $R_{j,t}$ are the factor prices in locally competitive factor markets.

Island j composite commodity is given by a CES aggregator of all island j firms' output:

$$Y_{j,t} = \left(\int_{\mathbb{I}} Y_{i,j,t}^{\frac{1}{1+\eta}} di\right)^{1+\eta},$$

where $\eta \geq 0$ controls the monopoly power. For simplicity, it is assumed that $\eta = 0$ so that island firms are perfect competitive. The competitive final-good sector employs a CES production function

$$Y_t = \left(\int_{\mathbb{J}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 0$ elasticity of substitution among island commodities, the inverse of which measures the strength of aggregate demand externalities.

Productivity and Ambiguity Shocks. Aggregate productivity $a_t \equiv \log A_t$ follows an AR(1) process

$$a_t = \rho a_{t-1} + \zeta_t,$$

where $\zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$ is the aggregate productivity shock in period t. Island-specific productivity $a_{j,t} \equiv \log A_{j,t}$ equals aggregate productivity plus an idiosyncratic productivity shock $\iota_{j,t}$:

$$a_{j,t} = a_t + \iota_{j,t}.$$

The idiosyncratic productivity shock $\iota_{j,t}$ is assumed to be i.i.d normally distributed with mean ω_t and variance σ_ι^2. Objectively, the cross-sectional mean of idiosyncratic productivity shocks are zero for all periods, that is, $\omega_t = 0 \ \forall t > 0$. However, DMs possess ambiguity over the cross-sectional means of idiosyncratic productivity shocks for all periods $t \geq 0$, that is, $\boldsymbol{\omega}_0 \equiv (\omega_0, \omega_1, \cdots, \omega_t, \cdots)$.[28]

At the beginning of period 0, all DMs have the common prior that all ω_t with $t \geq 0$ are i.i.d normally distributed with mean 0 and variance $\bar{\sigma}_\omega^2$, where $\bar{\sigma}_\omega^2$ measures the amount of ambiguity that DMs possess in the Amb.-SS. Ambiguity in the past does not last forever. As will become evident later, contemporaneous ambiguity is resolved at stage 2 of the same period. Therefore, at stage 0 of any period t, DMs inside the economy only possess ambiguity about contemporaneous and future cross-sectional means of idiosyncratic productivity shocks, that is, $\boldsymbol{\omega}_t \equiv (\omega_t, \omega_{t+1}, \cdots \omega_{t+k}, \cdots)$. It is assumed that DMs possess a common prior about $\boldsymbol{\omega}_t$ at stage 0 of period t such that ω_{t+k} with $k \geq 0$ are i.i.d normally distributed with

$$\omega_{t+k} \sim \mathcal{N}\big(0, \sigma^2_{\omega,t+k|t}\big) \quad \forall k \geq 0,$$

where $\sigma^2_{\omega,t+k|t}$ measure the amount of ambiguity that DMs perceived about ω_{t+k} at stage 0 of period t.

Denote $\bar{\psi} \equiv \bar{\sigma}_\omega^2/(\sigma_\xi^2 + \sigma_\iota^2 + \bar{\sigma}_\omega^2)$ and $\Psi_{t|t} \equiv \sigma^2_{\omega,t|t}/(\sigma_\xi^2 + \sigma_\iota^2 + \sigma^2_{\omega,t|t})$. We then assume that the log difference between Ψ_t and $\bar{\psi}$ follows an AR(1) process:

$$\widehat{\psi}_t \equiv \log(\Psi_{t|t}/\bar{\psi}) = \rho_\psi \widehat{\psi}_{t-1} + \tau_t \qquad\qquad \tau_t \sim \mathcal{N}\big(0, \sigma_\tau^2\big),$$

where τ_t is defined to be the ambiguity shock in our DSGE model. In the static model of Section 4, the ambiguity shock is defined to be the exogenous variation in $\sigma^2_{\omega,t}$. Observe that $\widehat{\psi}_t$ is a monotone transformation of $\sigma^2_{\omega,t|t}$. Hence, exogenous variations in $\widehat{\psi}_t$ can be regarded as a monotone transformation of exogenous variations in the amount of ambiguity $\sigma^2_{\omega,t|t}$. As will become evident later, formulating the ambiguity shock as exogenous variations in a specific monotone transformation of $\sigma^2_{\omega,t|t}$ instead of its original form greatly simplifies the numerical solution. Such a reverse engineering approach can also be found in Nimark (2014).

Denote $\Psi_{t+k|t} \equiv \sigma^2_{\omega,t+k|t}/(\sigma_\xi^2 + \sigma_\iota^2 + \sigma^2_{\omega,t+k|t})$ and $\widehat{\psi}_{t+k|t} \equiv \log(\Psi_{t+k|t}/\bar{\psi})$. We assume that $\widehat{\psi}_{t+k|t}$ is an increasing affine function of $\widehat{\psi}_t$:

$$\widehat{\psi}_{t+k|t} = \rho_\psi^k \widehat{\psi}_t.$$

Such a structure of prior ensures a certain notion of consistency in beliefs about all future ambiguity ω_{t+k}, $\forall k \geq 1$. It simply says that the period t prior over future cross-sectional means of idiosyncratic productivity shock ω_{t+k} coincides with the period $t + k$ prior if there were no ambiguity shocks between period t and $t + k$, that is, $\widehat{\psi}_{t+k|t} = \mathbb{E}_t[\widehat{\psi}_{t+k}]$.

28. In the dynamic model, we assume that the ambiguity is about the cross-sectional mean of idiosyncratic productivity shocks. It is isomorphic to the setup where DMs are ambiguous about the temporary component of the aggregate productivity shock.

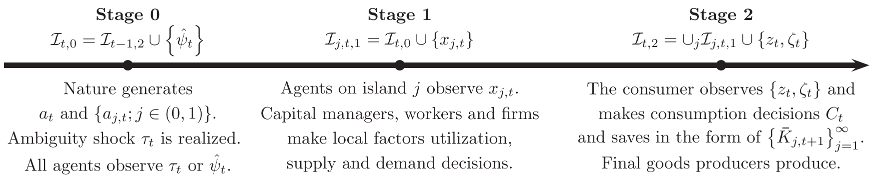

FIGURE 6. Timeline for period t of the DSGE model.

The ambiguity shock τ_t can be understood as a changing prior process. Moreover, we implicitly assume that an adverse ambiguity shock in period t, that is, some $\tau_t > 0$, makes DMs become more ambiguous about not only the contemporaneous cross-sectional mean of idiosyncratic productivity shock ω_t but also those of the entire future, that is, ω_{t+k} with $k \geq 1$. However, it is period t biased in the sense that it raises the perceived ambiguity contemporaneously more than those in the future. Finally, the increase in ambiguity is mean-reverting such that for ambiguity ω_{t+k} in the very far future $k \to +\infty$, the subjective belief stays in its Amb.-SS belief, that is, $\lim_{k\to+\infty} \widehat{\psi}_{t+k|t} = 0$ or $\lim_{k\to+\infty} \sigma^2_{\omega,t+k|t} = \bar{\sigma}^2_{\omega}$.

Information Structure. Denote the information sets (1) that are available to all DMs at stage 0 of period t, (2) that are only available to island j DMs at stage 1 of period t, and (3) that are available to all DMs at stage 2 of period t as $\mathcal{I}_{t,0}$, $\mathcal{I}_{j,t,1}$, and $\mathcal{I}_{t,2}$, respectively. Recursively, we can define these information sets by

$$\mathcal{I}_{t,0} = \mathcal{I}_{t-1,2} \cup \{\widehat{\psi}_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \{a_{j,t}\} \qquad \mathcal{I}_{t,2} = \cup_j \mathcal{I}_{j,t,1} \cup \{\omega_t\}.$$

Note that all contemporaneous uncertainty, either risk (state uncertainty over a_t) or ambiguity (model uncertainty over ω_t) is resolved at stage 2 of period t. Hence, consumption-saving decisions are made on the basis of complete information about future cross-sectional means of idiosyncratic shocks. Moreover, $\{a_{j,t}\}_{j\in\mathbb{J}}$ contains the same information about island j productivity in period $t+1$ as $\int_{\mathbb{J}} a_{j,t} dj$ does since idiosyncratic productivity shocks $\iota_{j,t}$ are i.i.d. Therefore, we can simplify the information set at stage 2 of period t by $\mathcal{I}_{t,2} = \mathcal{I}_{t,0} \cup \{\int_{\mathbb{J}} a_{j,t} dj, \omega_t\}$. To facilitate the notation, we further transform the information structure into

$$\mathcal{I}_{t,0} = \mathcal{I}_{t-1,2} \cup \{\psi_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \{x_{j,t}\} \qquad \mathcal{I}_{t,2} = \mathcal{I}_{t,0} \cup \{z_t, \zeta_t\},$$

where $x_{j,t} \equiv \zeta_t + \iota_{j,t}$ and $z_t \equiv \zeta_t + \omega_t$ denote the de-facto private information at stage 1 and public information at stage 2 about the aggregate productivity shock ζ, respectively. Figure 6 displays the timeline and information sets for period t in our dynamic model.

Preferences. Denote $s_{t+1} \equiv \mathcal{I}_{t+1,2} \backslash \mathcal{I}_{t,2}$ as the new information received by DMs at stage 2 between two consecutive periods t and $t+1$. We summarize the belief of the consumer at stage 2 by two corresponding Bayesian posteriors: (1) $\pi(s_{t+1}|\mathcal{I}_{t,2}, \boldsymbol{\omega}_t)$, the Bayesian posterior of s_{t+1} at stage 2 of period t under a particular model $\boldsymbol{\omega}_t$, and

(2) $\mu(\boldsymbol{\omega}_t|\mathcal{I}_{t,2})$, the Bayesian posterior of possible model $\boldsymbol{\omega}_t \in \mathcal{M}$, where $\mathcal{M}$ denotes the entire set of possible models.

The preference of the consumer at stage 2 of period t is represented by the recursive smooth model of ambiguity proposed by Klibanoff, Marinacci, and Mukerji (2009):

$$V_t(\mathcal{I}_{t,2}) = u(C_t - b\overline{C}_{t-1}) + \underbrace{\beta\varphi^{-1}\left(\int_{\mathcal{M}} \varphi\left(\int_{\mathcal{S}_{t+1}} V_{t+1}(\mathcal{I}_{t,2}, s_{t+1})\, d\pi\left(s_{t+1}|\mathcal{I}_{t,2}, \boldsymbol{\omega}_t\right)\right) d\mu\left(\boldsymbol{\omega}_t|\mathcal{I}_{t,2}\right)\right)}_{\text{Utility Equivalence of the Ambiguous Continuation Value}},$$

where $\varphi(\cdot)$ is some strictly increasing and concave function, whose curvature captures DMs' taste for ambiguity, namely the degree of ambiguity aversion.[29] The additional concavity in $\varphi(\cdot)$ captures the fact that an ambiguous continuation value reduces the utility of the DM because the ambiguity averse consumer dislikes the mean-preserving spread in the expected continuation value due to ambiguity.

Denote the value function as $J_t \equiv J(\{\overline{K}_{j,t}\}, a_{t-1}, z_t, \zeta_t, \psi_t)$. We then have the following Bellman equation for the consumer problem at stage 2:

$$J_t = \max_{C_t, \{I_{j,t+1}\}} u(C_t - b\overline{C}_{t-1}) + \beta\varphi^{-1}\left(\int_{\mathbb{R}} \varphi\left(\mathbb{E}_{t,2}^{\omega_{t+1}}[J_{t+1}]\right) f_t(\omega_{t+1}) d\omega_{t+1}\right) \tag{18}$$

subject to the budget constraint (15) and the physical capital accumulation equation (16). We assume that the consumer has external habit: $b \in (0, 1)$ controls for the degree of habit persistence, and $\overline{C}_{t-1}$ denotes the aggregate consumption.

Note that $\mathbb{E}_{t,2}^{\omega_{t+1}}[\cdot]$ stands for the mathematical expectation conditioned on $\mathcal{I}_{t,2}$ under a particular model ω_{t+1} for the cross-sectional mean of the idiosyncratic productivity shock tomorrow. And $f_t(\omega_{t+1})$ stands for the probability density function for the period t prior of ω_{t+1}, that is, the cross-sectional mean of the idiosyncratic productivity shock in period $t + 1$. Since period t knowledge does not reveal any information about ω_{t+1}, the prior belief about ω_{t+1} at stage 2 of period t coincides with that at stage 0.

29. Klibanoff, Marinacci, and Mukerji (2009) prove that if φ^{-1} is Lipschitz and the space of ambiguous model parameters is finite, the recursive smooth model of ambiguity will converge uniformly to the expected utility preferences with true model parameters. In our model, DMs are ambiguous over an infinite parameter space $\Omega_0 \equiv \{\omega_t : \forall t \geq 0\}$. This prevents the ambiguity from vanishing in the long run through learning.

At stage 1 of period t, the worker problem is given by (2) with the worker's posterior belief about possible models $\tilde{f}^{w}_{j,t,1}(\omega_t)$ given by:

$$\tilde{f}^{w}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}[J_t]\right)}{\varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t - b\bar{C}_t)}{P_t} W_{j,t} N_{j,t} - \frac{N^{1+\varepsilon}_{j,t}}{1+\varepsilon}\right]\right)}}_{\text{Weights}} \underbrace{f(a_{j,t}|\omega_t) f_t(\omega_t)}_{\text{Bayesian Kernel}}.$$

Similarly, the firm problem is formulated in a similar fashion as the static model without capital:

$$\max_{K_{i,j,t}, N_{i,j,t}} \int_{\Omega_t} \varphi\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t - b\bar{C}_t)}{P_t} \Pi_{i,j,t}\right]\right) \tilde{f}^{f}_{j,t,1}(\omega_t) d\omega_t,$$

where $\Pi_{i,j,t}$ is given by (17) and firms' posterior belief about possible models $\tilde{f}^{f}_{j,t,1}(\omega_t)$ is such that:

$$\tilde{f}^{f}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}[J_t]\right)}{\varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t - b\bar{C}_t)}{P_t} \Pi_{i,j,t}\right]\right)}}_{\text{Weights}} \underbrace{f(x_{j,t}|\omega_t) f_t(\omega_t)}_{\text{Bayesian Kernel}}.$$

Finally, the capital manager seeks to maximize the consumer valuation of the proceeds from managing physical capital and the manager problem is such that

$$\max_{u_{j,t}} \int_{\mathbb{R}} \varphi\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t - b\bar{C}_t)}{P_t} R_{j,t} u_{j,t} \bar{K}_{j,t} - \Xi(u_{j,t}) \bar{K}_t\right]\right) \tilde{f}^{cm}_{j,t,1}(\omega_t) d\omega_t,$$

where capital manager's posterior belief about possible models $\tilde{f}^{cm}_{j,t,1}(\omega_t)$ is such that:

$$\tilde{f}^{cm}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\varphi'\left(\mathbb{E}^{\omega_t}_{t,0}[J_t]\right)}{\varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t - b\bar{C}_t)}{P_t} R_{j,t} u_{j,t} \bar{K}_{j,t} - \Xi(u_{j,t}) \bar{K}_t\right]\right)}}_{\text{Weights}} \underbrace{f(x_{j,t}|\omega_t) f_t(\omega_t)}_{\text{Bayesian Kernel}}.$$

Note that the worker's, the firm's, and the manager's posterior beliefs all follow the extended smooth rule of updating to ensure dynamic consistency from the consumer's perspective. Furthermore, our formulation ensures that the stochastic discount factor that the households use to evaluate factor payments is given by $(U'(C_t - b\bar{C}_t)/P_t)\varphi'(\mathbb{E}^{\omega_t}_{t,0}[J_t])$, which does not only take care of households' risk attitude $U'(C_t - b\bar{C}_t)/P_t$ but also their ambiguity attitude $\varphi'(\mathbb{E}^{\omega_t}_{t,0}[J_t])$.

Market Clearing and GDP. The market clearing conditions for labor and effective capital are such that

$$N_{j,t} = \int_{\mathbb{I}} N_{i,j,t} di \qquad u_{j,t} \bar{K}_{j,t} = \int_{\mathbb{I}} K_{i,j,t} di.$$

The resource constraint is such that

$$Y_t = C_t + \int_{\mathbb{J}} I_{j,t} dj + G_t + \int_J \Xi(u_{j,t}) \bar{K}_{j,t} dj,$$

where G_t denotes public spending, which is exogenously determined as a time-varying fraction of output

$$G_t = gY_t.$$

And finally, GDP is defined to be[30]

$$Q_t = C_t + \int_{\mathbb{J}} I_{j,t} dj + G_t.$$

To close up the description of the model, we make the following functional form assumption:

ASSUMPTION 3 (*Log-Exponential*). We assume log utility and CAAA:

$$u(C_t) = \ln C_t, \qquad \varphi(x) = -\frac{1}{\lambda} \exp(-\lambda x) \quad \text{with } \lambda \geq 0.$$

In Online Appendix F.1, we list out the set of optimality conditions. The quantitative methodology, which is closely related to what we have done in Section 5.1, is discussed in Online Appendix F.2. The detailed step-by-step guidance about the solution method can be found in Online Appendix F.3. Finally, we analyze the accuracy of our solution method in Online Appendix F.4.

6.2. Calibration and Estimation

Table 3 summarizes the estimated parameters used in our baseline model. There are three sets of parameters. The first set of parameters is calibrated following a standard DSGE exercise. Specifically, to stay close to the existing DSGE literature, we choose the discount factor β to be 0.99; the Frisch elasticity of labor supply to be 2; the capital share in production to be 0.3, the depreciation rate of capital to be 0.015, and finally, and the share of government expenditure in output to be 0.2. The persistence

30. Without variable utilization, Q_t and Y_t are identical. With variable utilization, the gap between the two would be the real cost of variable utilization $\int_J \Xi(u_{j,t}) \bar{K}_{j,t} dj$. Following Justiniano, Primiceri, and Tambalotti (2010) and Angeletos, Collard, and Dellas (2018), when we bring our DSGE model to the data, we map Q_t to output in the data.

TABLE 3. Model parameters.

Parameters	Role	Value
	Calibrated parameter	
β	Discount factor	0.99
ε	Frisch elasticity of labor supply (inverse)	0.5
α	Capital share	0.3
δ	Depreciation rate	0.015
g	Share of government expenditure	0.2
ρ	Persistence of aggregate productivity shock	0.89
$100\sigma_{\zeta}$	Standard deviation of aggregate productivity	0.34
$100\sigma_{\tau}$	Standard deviation of ambiguity shock	0.93
	Estimated parameters (matching IRFs)	
σ_{ι}	Standard deviation of idiosyncratic productivity shock	0.1414
θ	Degree of aggregate demand externality (inverse)	0.1750
λ	Degree of ambiguity aversion	11.04
$\overline{\psi}$	Amount of ambiguity at Amb.-SS	0.8215
b	Parameter of habit formation	0.9315
φ	Parameter of investment adjustment cost	0.6605
ξ	Parameter of variable capital utilization	0.4106
ρ_{ψ}	Persistence of ambiguity shock	0.7988

and standard deviation of the aggregate productivity shock are inferred from Fernald (2014): $\rho = 0.95$ and $100\sigma_{\zeta} = 0.568$. Finally, the standard deviation of the ambiguity shock is set to $100\sigma_{\tau} = 0.93$, which equals the standard deviation of the PD shock identified in the empirical VAR exercise of Section 3.

The second set of parameters, including the standard deviation of the idiosyncratic productivity shock (σ_{ι}), the inverse degree of aggregate demand externality (θ), the degree of ambiguity aversion (λ), the amount of ambiguity at the Amb.-SS ($\overline{\psi}$), parameters that control the habit formation (b), the investment adjustment cost (φ) and the variable capital utilization (ξ), and finally the persistence of the ambiguity shock (ρ_{ψ}), are estimated by matching the theoretical IRFs with their empirical counterparts in a similar fashion as Christiano, Eichenbaum, and Evans (2005) and Altig et al. (2011). Specifically, these parameters are chosen to minimize the distance between theoretical IRFs of output, consumption, hours, investment, pessimism, and disagreement to the ambiguity shock and their empirical counterparts, namely the empirical IRFs of the above variables to the identified PD shock. We include the first 20 moments weighted by the precisions of the empirical moments. Our estimation strategy can be justified by the fact that ambiguity shock in our theoretical framework is the only shock that generates simultaneous variations in pessimism and disagreement. Therefore, if there is a shock that explains the maximal volatility in both pessimism and disagreement, which is how we identify the PD shock, it must be the ambiguity shock.

Figure 7 demonstrates the matching between the IRFs to the theoretical ambiguity shock and those to the empirical PD shock. The estimated parameters succeed in capturing the dynamic co-movements among pessimism, disagreement, and real quantities, including the hump-shaped response in real quantities as well as counter-cyclical movements in pessimism and disagreement. In a standard log-linearized

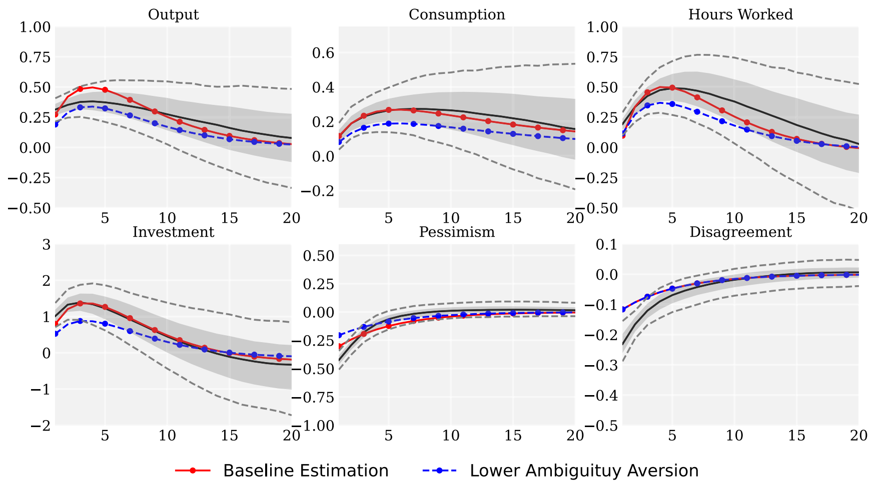

FIGURE 7. IRFs to the theoretical ambiguity shock and to the empirical PD shock. This figure plots the impulse response functions of output, consumption, hours, investment, pessimism, and uncertainty to the theoretical ambiguity shock (red solid line with dots for baseline estimation with $\lambda = 11.04$ and blue dashed line with dots for $\lambda = 7.73$) and the empirical PD shock (black solid line). Horizontal axis: time horizon in quarters. Shaded area: 68% HPDI for the empirical IRFs of the PD shock. Gray dashed lines: 95% HPDI for the empirical IRFs of the PD Shock.

DSGE model, the standard deviation of a shock is independent of policy rules and has no impact on the shape of IRFs. However, with information frictions, this is never the case. In our baseline, the estimated standard deviation of idiosyncratic productivity shock is 0.1414. It is very close to that of Straub and Ulbricht (2018), which is calibrated to be broadly consistent with empirical estimates using plant-level data. Furthermore, notice that the persistence of the ambiguity shock is 0.7988, at which the half-life of the ambiguity shock is close to that of the aggregate demand shock identified in Blanchard and Quah (1989). Moreover, the estimated degree of ambiguity aversion is 11.04. It is very close to that calibrated in Collard et al. (2018), which is shown to be consistent with asset pricing evidence.

Note that in the empirical IRFs to the PD shock, pessimism and disagreement are measured using data on households' forecasts on the unemployment rate. However, in our DSGE model, there is no comparable notion of the unemployment rate since the labor market is frictionless, whereas, in our theoretical IRFs, we use quarter-ahead output forecasts as the theoretical counterparts. Such a choice is without loss of generality since unemployment and output are highly correlated within the business cycles and hence are also their forecasts.

6.3. *Ambiguity Shock as Non-Inflationary Aggregate Demand Shock*

The success of our DSGE model in matching empirical IRFs to PD shock hinges upon the fact that the ambiguity shock in our theory acts as a non-inflationary aggregate demand shock, which drives the positive co-movements across real quantities without

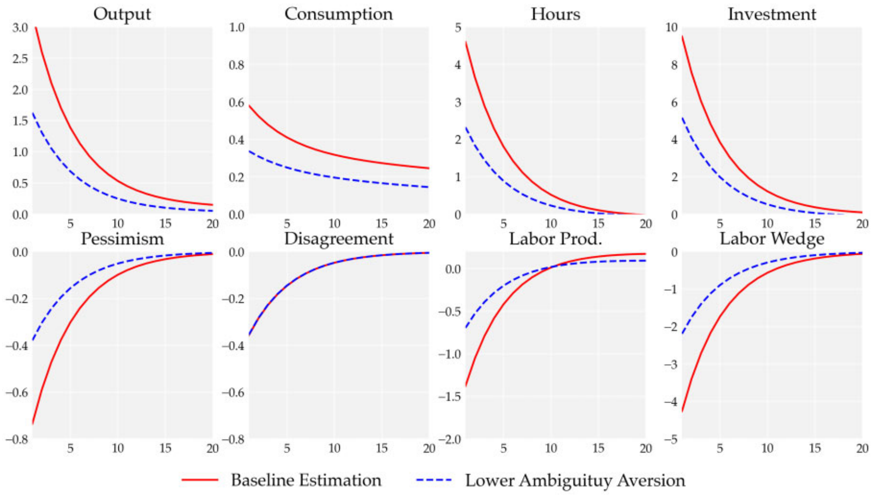

FIGURE 8. Propagation mechanism of the ambiguity shock: IRFs. This figure plots the impulse response functions of output, consumption, hours, investment, pessimism, and uncertainty to theoretical ambiguity shock by shutting down "bells-and-whistles" of DSGE model. Blue solid line: benchmark estimation with $\lambda = 11.04$. Red dashed line: lower degree of ambiguity aversion $\lambda = 7.73$. Horizontal axis: time horizon in quarters.

relying on the NKPC as the propagation mechanism. To demonstrate such an empirical property of the ambiguity shock, we momentarily shut down all "bells and whistles" of our DSGE model, including habit formation, investment adjustment cost, and variable capital utilization by setting $b = \varphi = 0$ and $\xi = 1$. Figure 8 reports the IRFs of key macro statistics to an expansionary ambiguity shock, that is, a shock that decreases the amount of ambiguity perceived by all DMs of the economy. An expansionary ambiguity shock generates simultaneous booms in real quantities, that is, output, consumption, hours, and investment, while, at the same time, decreases the labor productivity and the labor wedge.[31]

The driving force of dynamic co-movements behind the above IRFs is the fluctuations in the degree of pessimism and the existence of incomplete information. The former generates variations in agents' expectation about the outlook of the economy. And the latter breaks the Barro–King critique by moving labor choices prior the common knowledge of aggregate productivity as in Ilut and Saijo (2021) and Angeletos and Lian (2021), which is the primary reason why there are co-movements across real quantities. Additionally, the fluctuations in the degree of pessimism is mostly about the short-run outlook of the economy instead of the long-run. It limits the strength of wealth effect making substitution effect dominates in equilibrium, which contributes the co-movements across real quantities. Specifically, an expansionary

31. See Angeletos, Collard, and Dellas (2018) for a detailed discussion in the construction of the labor wedge.

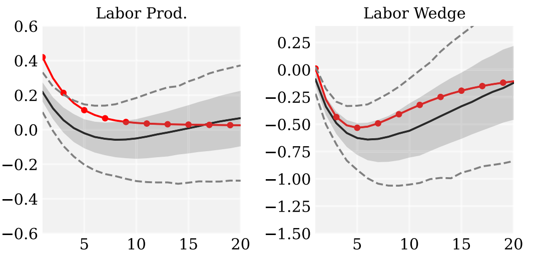

FIGURE 9. IRFs of labor productivity and labor wedge. This figure plots the impulse response functions of labor productivity and labor wedge to the theoretical ambiguity shock (red solid line with dots) and the empirical PD shock (black solid line). Horizontal axis: time horizon in quarters. Shaded area: 68% HPDI for the empirical IRFs of the PD shock. Dashed lines: 95% HPDI for the empirical IRFs of the PD shock.

ambiguity shock, by construction, weakens the degree of pessimism of all DMs about the cross-sectional mean of idiosyncratic productivity shocks not only for today but also for the future. From the firm perspective, such an increased pessimism means a cheerful expectation about aggregate demand. In response, firms expand their demand for labor and capital, generating upward pressures on factor prices. From the household perspective, it then implies an increase in expected permanent income, making consumption goes up. At the same time, the household also understands that the upward pressures on factor prices only last in the short run, which restricts the strength of the wealth effect. Therefore, hours and investment also increase in equilibrium, since the relevant substitution effect dominates the opposing wealth effect.

The way in which the ambiguity shock creates aggregate dynamic co-movements resembles the aggregate demand shock in the NK literature, but without relying on the NKPC as the propagation mechanism. Unlike aggregate demand shocks in the NK framework, ambiguity shock can generate co-movement across real quantities even under flexible price allocations, which is consistent with the interpretation of the non-inflationary aggregate demand shock. The propagation mechanism of the ambiguity shock is akin to those of the confidence shocks in Angeletos, Collard, and Dellas (2018), Huo and Takayama (2015), and Ilut and Saijo (2021). Whereas, ambiguity also creates counter-cyclical movements in both pessimism and disagreement, a pattern that is consistent with survey data evidence.

The IRFs of the full DSGE model reported in Figure 7 are nothing more than the twisted version of IRFs in Figure 8. Habit formation and investment adjustment costs create hump-shaped responses of output, consumption, hours, and investment. Furthermore, variable capital utilization mitigates the dynamic responsiveness of hours to the ambiguity shock relative to that of output by creating a mild pro-cyclical movement in labor productivity. Finally, these "bells-and-whistles" also generate the empirically relevant IRFs of the labor wedge both in terms of counter-cyclicality and U-shaped dynamic responses (Figure 9).

Role of Ambiguity Aversion. Ambiguity aversion is crucial for the propagation mechanism of the ambiguity shock. If agents are ambiguity neutral $\lambda = 0$, there would be no pessimistic belief about the outlook of the economy. As a consequence, the ambiguity shock creates no fluctuations in pessimism, which results in zero variations in real quantities. Unlike pessimism and real quantities, the way how the ambiguity shock maps into fluctuations in disagreement is unaffected by the degree of ambiguity aversion. A natural question then arises. To what extent does the estimated degree of ambiguity aversion matter for the quantitative results of our model? To answer this question, we compare the model implications of our baseline estimation with those of a lower degree of ambiguity aversion. Specifically, we consider an alternative model, in which the degree of ambiguity aversion equals 70% of that under the baseline estimation: $\lambda = 11.05 \times 0.7 = 7.73$. According to equations (F.1) and (F.2) of Online Appendix F.2, a reduction in the degree of ambiguity aversion λ results in a reduced degree of pessimism about contemporaneous and future cross-sectional mean of idiosyncratic productivity shock, that is, $\widetilde{\mathbb{E}}_{j,t,1}[\omega_t]$ and $\widetilde{\mathbb{E}}_{t,2}[\omega_{t+1}]$, which eventually maps into reduced responses of pessimism and real quantities to the ambiguity shock. Moreover, because the use of private information is unaffected by the amount of ambiguity as suggested by Proposition 14 or equation (F.1), the response of disagreement to the ambiguity shock is unaffected by a lower degree of ambiguity aversion. The way how the reduction in the degree of ambiguity aversion affects the dynamic co-movement patterns or IRFs of the ambiguity shock is robust with respect to the existence of the "bells-and-whistles" of DSGE model (Figure 7 for the full DSGE and Figure 8 for the RBC). Similar image arises when we look at the business-cycle moments by comparing Column II and Column III of Table 4: a lower degree of ambiguity aversion results in lower volatilities in pessimism and real quantities and has zero impact on the volatility of disagreement.

6.4. Business-Cycle Moments

Table 4 summarizes the business-cycle moments of real quantities in the US data throughout 1985Q1-2017Q4 (Column I) and our estimated DSGE model (Column II). The empirical fit of our estimated DSGE model is satisfactory. It captures the fact that real quantities co-move positively with each other at the business-cycle frequencies. In addition, our estimated DSGE model can account for the bulk of business-cycle volatilities in real quantities, including output (88%), consumption (40%), hours (55%), investment (64%), labor productivity (78%), and labor wedge (41%). Besides, estimated DSGE model succeeds in generating a weak correlation between output (q) and labor productivity (y/h) and between hours (h) and labor productivity (y/h). Our estimated DSGE model tends to over-predict the correlations of output and hours with labor productivity. However, the data counterparts of the two correlations are not stable over time. If we extend the aggregate time series for real quantities from 1985:I–2017:IV to 1955:I–2017:IV, the correlations of output and hours with labor productivity are 0.47 and 0.11, respectively, which are very close to the predictions of

TABLE 4. Business-cycle moments.

	Data (1985:I–2017:IV)	Baseline	Lower λ
Standard deviations			
$\sigma(q)$	0.93	0.82 (87.9%)	0.68 (72.8%)
$\sigma(c)$	0.72	0.29 (39.9%)	0.21 (29.3%)
$\sigma(n)$	1.41	0.78 (54.9%)	0.65 (45.6%)
$\sigma(i)$	4.10	2.60 (63.7%)	2.09 (51.2%)
$\sigma(y/n)$	0.73	0.57 (77.8%)	0.50 (68.6%)
$\sigma\left(\Delta_n\right)$	1.94	0.79 (40.9%)	0.67 (34.5%)
$\sigma(P)$	0.45	0.29 (65.4%)	0.20 (44.5%)
$\sigma(D)$	0.32	0.11 (35.4%)	0.11 (35.5%)
Correlations			
$\rho(c,q)$	0.88	0.88	0.84
$\rho(n,q)$	0.86	0.85	0.81
$\rho(i,q)$	0.97	0.99	0.99
$\rho(c,n)$	0.82	0.85	0.89
$\rho(c,i)$	0.82	0.82	0.79
$\rho(i,n)$	0.88	0.83	0.79
$\rho(q,P)$	−0.44	−0.55	−0.45
$\rho(q,D)$	−0.71	−0.55	−0.45
Correlation with productivity			
$\rho(q,y/n)$	0.04	0.63	0.65
$\rho(n,y/n)$	−0.45	0.17	0.13
Correlation with labor wedge			
$\rho(q,\Delta_n)$	−0.79	−0.54	−0.43
$\rho(c,\Delta_n)$	−0.84	−0.70	−0.61
$\rho(n,\Delta_n)$	−0.98	−0.89	−0.87
$\rho(i,\Delta_n)$	−0.80	−0.48	−0.38

Note: The first column reports the relevant business-cycle moments for US data from 1985:I to 2017:IV. The second column reports the relevant business-cycle moments of our DSGE model under baseline estimation ($\lambda = 11.04$). The third column reports the relevant business-cycle moments of our DSGE model with a lower degree of ambiguity aversion ($\lambda = 7.73$). All moments are band-pass filtered at frequencies of 6–32 quarters. The number in the brackets after standard deviation in the baseline model provides information about what percentage of volatility in the data that can be explained by our baseline model.

our estimated DSGE model. The success of our DSGE model in matching the business-cycle moments hinges upon the fact that it can generate the empirically relevant counter-cyclical variations in labor wedge (Δ_n), that is, negative correlations with other real quantities, including output, consumption, hours, and investment. Moreover, our DSGE model can also generate the empirically plausible business-cycle moments in both pessimism (P) and disagreement (D). Our estimated DSGE model captures 65% volatility of pessimism and 35% volatility in disagreement. Also, it successfully captures the counter-cyclicality of both pessimism and disagreement over the business-cycle frequencies.

6.5. *Ambiguity Shock as the Dominant Shock of Business Cycles*

Reproducing Interchangeability. To what extent can the ambiguity shock serve as the dominant shock of business cycles within our estimated DSGE model? Or equivalently,

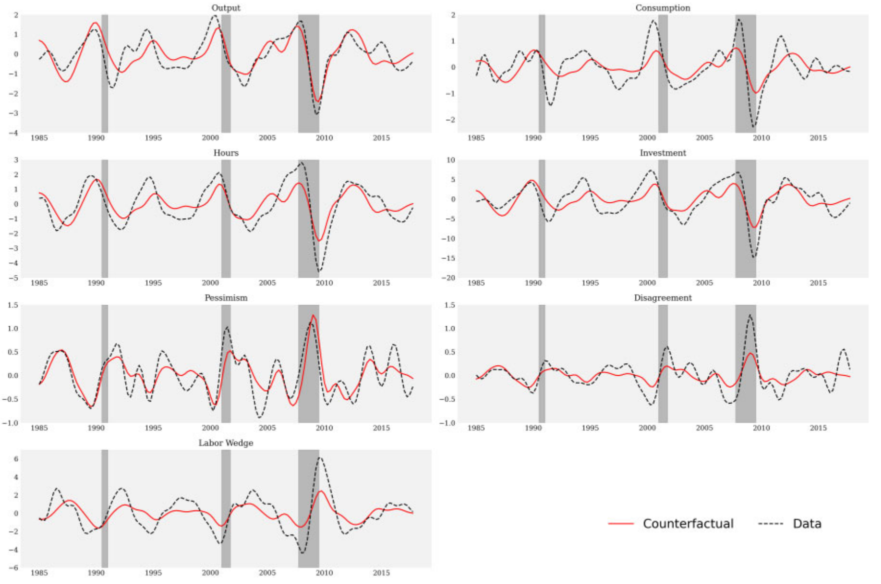

FIGURE 11. Data and counterfactual economy (only the ambiguity shock). Black dashed line: cyclical components pessimism, disagreement, real quantities, and labor wedge of the data from 1985:I to 2017:IV. Red solid line: cyclical components of pessimism, disagreement, real quantities, and labor wedge of the simulated counterfactual economy with only ambiguity shock from 1985:I to 2017:IV. All time series are band-pass filtered at frequencies 6–32 quarters.

to what extent can our estimated DSGE model reproduce the type of interchangeability that is identified in our empirical analysis? To answer these questions, we simulate our estimated DSGE model for $N = 1,000$ times with random initial conditions. In each of the simulation, we generate an artificial dataset that mimics the empirical dataset used in our empirical analysis. Specifically, each artificial dataset consists of 6 artificial time-series of the same length as in the data (132 quarters from 1985:I to 2017:IV), including output, consumption, hours, investment, pessimism, and disagreement. We randomize the initial conditions by simulating 232 observations and dropping the first 100. For each of the 1,000 artificial datasets, we then re-run our empirical analysis[32] and use the same identification strategy to identify the artificial PD shock and its many "cousin" shocks. Our model is able to reproduce the interchangeability as depicted by Figure 10. Furthermore, these IRFs are also indistinguishable from those to the theoretical ambiguity shock (Figure D.1 of Online Appendix D) or those to the empirical PD shock (Figure D.2 of Online Appendix D). It then suggests that, at the background of such interchangeability, it is the ambiguity shock acting as the dominant

32. When we apply our empirical strategy to artificial data, we run a smaller VAR consists of output, consumption, hours, investment, pessimism, and disagreement only. In addition, to improve the efficiency of the exercise, these VARs are estimated by the classical inference instead of the Bayesian inference.

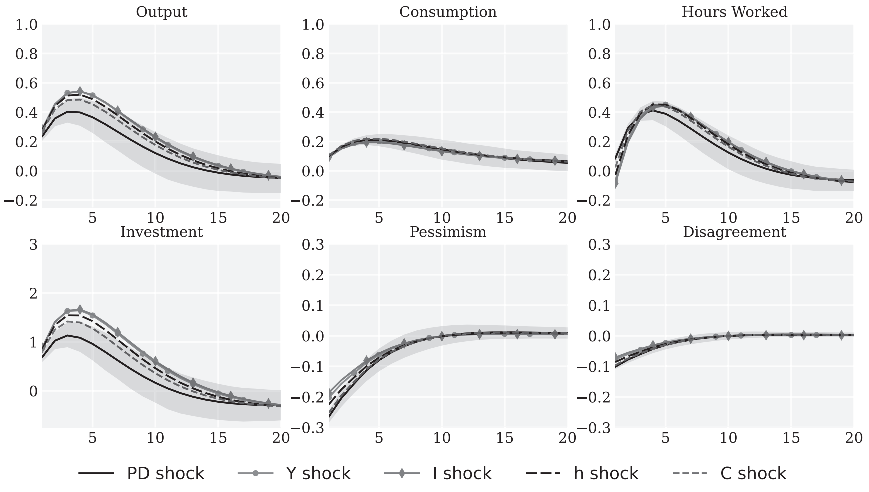

FIGURE 10. IRFs to the artificial PD shock and its many "cousin" shocks, the DSGE model. This figure plots the impulse response functions of all variables to the simulated PD shock and its many "cousin" shocks of our DSGE model. To generate these IRFs, we simulate our estimated DSGE model for 1,000 times. In each of the simulation, we generate an artificial dataset with the same length as the data consisting of output, consumption, hours, investment, pessimism, and disagreement. We then re-run our empirical analysis for each of the 1,000 artificial datasets and report the mean of the relevant IRFs in this figure. Horizontal axis: time horizon in quarters. Shaded area: 68% HPDI of the artificial PD shock.

shock in driving fluctuations in pessimism, disagreement, and real quantities.[33] Finally, the interchangeability extends to variance contributions of the artificial PD shock and its many "cousin" shocks (Table D.1 of Online Appendix D), which further demonstrates that the ambiguity shock serves as the dominant shock in driving the business cycles of pessimism, disagreement, and real quantities.

Historical Variance Contributions. To what extent does the ambiguity shock alone explain business-cycle fluctuations in pessimism, disagreement, and real quantities historically? To answer this question, we construct a counterfactual economy where there are only ambiguity shocks in our estimated DSGE model. We simulate the economy from 1985:I to 2017:IV. To start the simulation, we set consumption $\hat{c}_0$ and investment $\hat{i}_0$ in 1984:IV to their data counterparts, that is, the cyclical components of the two after applying a band-pass filter at frequencies 6–32 quarters. Physical capital $\hat{\hat{k}}_1$ is set equal to $\delta\hat{i}_0$ and TFP in 1984:IV is assumed to be at the steady-state level, $\hat{a}_0 = 0$.[34] Then, we feed our estimated DSGE model with a time series of the ambiguity

33. The indistinguishability between the artificial PD shock and the theoretical ambiguity shock in terms of IRFs also justifies our interpretation of the empirical PD shock as an structural shock instead of the footprints of multiple structural shocks through a common propagation mechanism.

34. The simulated counterfactual economy is robust to the choice of $\hat{\hat{k}}_1$ and $\hat{a}_0 = 0$.

TABLE 5. The comparison between data and counterfactual economy (1985:I–2017:IV).

	Output	Consumption	Hours	Investment	Pessimism	Disagreement	Labor wedge
$\sigma_{X,\mathrm{CF}}/\sigma_{X,\mathrm{DATA}}$	86%	48%	59%	59%	78%	42%	42%
$\mathrm{corr}(X_{\mathrm{CF}}, X_{\mathrm{DATA}})$	0.79	0.70	0.84	0.78	0.81	0.81	0.81

Note: The first row reports the standard deviations of real quantities, pessimism, and uncertainty in the counterfactual economy relative to those in the data. The second row reports the correlation between the simulated time series of the counterfactual economy and the actual time series of the data. All time series are band-pass filtered at frequencies of 6–32 quarters. The counterfactual economy features the ambiguity shock only.

shock that equals the empirically estimated time series of the PD shock from 1985:I to 2017:IV. It is assumed that from 1985:I onward the counterfactual economy features no variations in TFP, that is, the realized TFP shocks are always 0. To simulate the counterfactual economy, what remains to be specified is the amount of ambiguity in 1984:IV $\widehat{\psi}_0$. We calibrate it to perfectly match the cyclical component of pessimism at 1985:I in our simulated counterfactual economy.

Figure 11 compares the simulated time series for pessimism, disagreement, and real quantities in the counterfactual economy with their data counterparts. The bulk of the business-cycle variations in real quantities, pessimism, and disagreement can be explained by the ambiguity shock alone. Table 5 reports the standard deviations of pessimism, disagreement, and real quantities in the counterfactual economy relative to those in the data (Row 1). Between 1985:I and 2017:IV, ambiguity shock alone can explain 86% of the variations in output, 48% of the variations in consumption, 59% of the variations in hours, 59% of the variations in investment, 78% of the variations in pessimism, 42% of the variations in disagreement, and finally 42% of the variations in labor wedge. Row 2 of Table 5 further reports the correlation between the simulated time series of the counterfactual economy and the actual time series of the data. These correlations are about 0.80 except for 0.70 for consumption. It suggests that the ambiguity shock in our DSGE model can be regarded as the dominant shock or propagation mechanism for the business-cycle fluctuations in pessimism, disagreement, and real quantities.

7. Conclusion

This paper provides empirical evidence that the data admits a dominant shock that accounts for the bulk of fluctuations not only in real quantities but also in certain characteristics of households' subjective beliefs: the degree of pessimism relative to the rational expectations benchmark (pessimism) and the cross-sectional dispersion of beliefs (disagreement). The empirical evidence also suggests that the dominant shock is in the form of a non-inflationary aggregate demand shock given the fact that it is disconnected from either TFP or inflation at all frequencies.

We develop a theory of ambiguity-driven business cycles, where the ambiguity shock can generate positive co-movements across real quantities under the RBC framework without relying on the NKPC as the propagation mechanism. At the same time, the ambiguity shock can generate counter-cyclical pessimism and disagreement. Through the lens of an estimated DSGE model featuring flexible prices and a rich set of "bells-and-whistles", our theory reproduces the salient features of the macroeconomic data extended with data on households' expectations. Quantitatively, the ambiguity shock alone accounts for a significant fraction of the business-cycle volatility in pessimism, disagreement, and real quantities. It serves as the dominant non-inflationary aggregate demand shock that drives the bulk of the business-cycle fluctuations in pessimism, disagreement, and real quantities.

Appendix: Derivations and Proofs

Derivation of Equation (10)

The first order condition for the island j workers' problem is such that

$$\int_{\mathbb{R}} \varphi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t}W_{j,t}N_{j,t} - \chi\frac{N^{1+\varepsilon}_{j,t}}{1+\varepsilon}\right]\right) \times \mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{U'(C_t)}{P_t}W_{j,t} - \chi N^{\varepsilon}_{j,t}\right]\tilde{f}^{w}_{j,t,1}(\omega_t)d\omega_t = 0.$$

Plugging in the expression for $\tilde{f}^{w}_{j,t,1}(\omega_t)$ given by (3), we arrive at (7) where the distorted posterior belief about possible models can be given by (9). Similar procedures lead to (8). Then in the last step, combining (7) and (8) l (10). □

LEMMA A.1. *The distorted posterior belief $\tilde{f}_{j,t,1}(\omega_t)$ is Gaussian if allocation $\{\{Y_{j,t}\}_{j\in\mathbb{J}}, Y_t\}$ constitutes a conditional Log-Normal equilibrium.*

Proof. Under conditional log-normal equilibrium, we have that

$$\begin{aligned} y_{j,t} &= y^* + \bar{h}_y + \kappa_{ya,t}a_{j,t} + \hat{h}_y(\psi_t), \\ n_{j,t} &= n^* + \bar{h}_n + \kappa_{na,t}a_{j,t} + \hat{h}_n(\psi_t), \\ y_t &= \int_{\mathbb{J}} y_{j,t}dj + \frac{1}{2}\left(1-\frac{1}{\theta}\right)d^2_{y,j}, \end{aligned}$$

where $d_{y,j} \equiv \kappa^2_{ya,t}\sigma^2_\iota$ denotes the cross-sectional dispersion of island outputs. We ignore $d_{y,j}$ in the approximation without loss of generality since it is of second-order impacts at the aggregate level and have no impact at all on the cyclical behavior of various dispersion measures.

Define $\bar{S} \equiv e^{s^*+\bar{h}_s}$ for any variable of interest S, which denotes the level of variable S at the Amb.-SS. It is then straight-forward to show that

$$U(Y_t) = y^* + \bar{h}_y + \hat{y}_t.$$

Further define stage 0 ex-ante expected utility under a particular model ω_t as $\bar{J}_t(\omega_t) \equiv \mathbb{E}^{\omega_t}_{t,0}[U(Y_t)]$, which is given by

$$\bar{J}_t(\omega_t) = \mathrm{Const}_t + \kappa_{ya_j}(\psi_t)\omega_t.$$

The expression of $\bar{J}_t(\omega_t)$ implies that the belief distortion in (12) is of exponential quadratic form. Therefore, the posterior belief about possible models $\tilde{f}_{j,t,1}(\omega_t)$ is normal. With a bit algebra, it can be shown that $\tilde{f}_{j,t,1}(\omega_t)$ is a normal density with the mean $\mu_{j,t}$ and the variance σ_t^2 given by

$$\mu_{j,t} = \left(\frac{e^{\psi_t}}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) a_{j,t} + \left(\frac{\sigma_\xi^2 + \sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) g_\mu(\psi_t, \lambda) \tag{A.1}$$

and

$$\sigma_t^2 = \left(\frac{\sigma_\xi^2 + \sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) e^{\psi_t}, \tag{A.2}$$

where the distortion in mean is $g_\mu(\psi_t, \lambda) = -\lambda\kappa_{ya_j}(\psi_t)e^{\psi_t}$. □

LEMMA A.2. *Allocation* $\{\{Y_{j,t}(a_{j,t}, \psi_t)\}_{j\in\mathbb{J}}, Y_t(a_t, \psi_t)\}$ *constitutes a conditional log-normal equilibrium if distorted posterior belief over possible models* $\tilde{f}_{j,t,1}(\omega_t)$ *is Gaussian.*

Proof. Directly follows Angeletos and La'O (2009). □

Proof of Proposition 1

Suppose that the conditional log-normal equilibrium is such that:

$$y_{j,t} \equiv \ln Y_{j,t} = y^* + \bar{h}_y(\bar{\psi}, \lambda) + \kappa_{ya_j}(\psi_t)a_{j,t} + \hat{h}_y(\psi_t, \lambda)$$
$$y_t \equiv \ln Y_t = y^* + \bar{h}_y(\bar{\psi}, \lambda) + \kappa_{ya_j}(\psi_t)a_t + \hat{h}_y(\psi_t, \lambda),$$

where we ignore the dispersion adjustment of aggregate output in the approximation without loss of generality since they are of second-order impacts at the aggregate and have no impacts at all on the various dispersion measures. Then at D-SS, we have the following:

$$\ln(\chi) + \left(\frac{1+\varepsilon}{1-\alpha}\right) y^* = \ln(1-\alpha).$$

While, at Amb.-SS, the impacts of the ambiguity shock at Amb.-SS denoted by $\bar{h}_y$ must satisfy the following:

$$\left(\frac{1+\varepsilon}{1-\alpha}\right) \bar{h}_y = \left(\frac{1}{\theta} - 1\right) H_y(\bar{\psi}, \lambda),$$

where $H_y(\bar{\psi}, \lambda)$ denotes the degree of pessimism of island j DMs' concerning aggregate output y_t at the Amb.-SS.[35] Under the proposed conditional log-normal equilibrium, it is given by

$$H_y(\bar{\psi}, \lambda) = \kappa_{ya_j}(\bar{\psi}) \left(\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}[a_t] \tilde{f}_{j,t,1}(\omega_t) d\omega_t \right)\Bigg|_{a_{j,t}=0, \psi_t = \bar{\psi}}.$$

Following (A.1), we have that

$$H_y(\bar{\psi}, \lambda) = \kappa_{ya_j}(\bar{\psi}) \left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\bar{\psi}}} \right) g_\mu(\bar{\psi}, \lambda),$$

where $\kappa_{ya_j}(\bar{\psi})$ denotes the use of private information at the Amb.-SS.

In the next step, we log-linearize (11) around the Amb.-SS:

$$\left(\frac{1+\varepsilon}{1-\alpha} - 1 + \frac{1}{\theta}\right) \hat{y}_{j,t} = \left(\frac{1+\varepsilon}{1-\alpha}\right) a_{j,t} + \left(\frac{1}{\theta} - 1\right) \left(\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}\left[\hat{y}_t\right] \tilde{f}_{j,t,1}(\omega_t) d\omega_t - H_y(\bar{\psi}, \lambda) \right).$$

Matching coefficients lead to the two following equilibrium conditions

$$\left(\frac{1+\varepsilon}{1-\alpha}\right) \kappa_{ya_j}(\psi_t) = \left(\frac{1+\varepsilon}{1-\alpha}\right) - \left(\frac{1}{\theta} - 1\right) \left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}} \right) \kappa_{ya_j}(\psi_t), \quad \text{(A.3)}$$

$$\left(\frac{1+\varepsilon}{1-\alpha}\right) \hat{h}_y(\psi_t, \lambda) = \left(\frac{1}{\theta} - 1\right) \left(H_y(\psi_t, \lambda) - H_y(\bar{\psi}, \lambda) \right). \quad \text{(A.4)}$$

35. To understand why this is the case, recall that the Amb.-SS refers to the state into which the economy converges (a) in the absence of any shocks, that is, $a_{j,t} = 0$, but (b) taking into account the existence of ambiguity, that is, evaluating $\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}\left[a_t\right] \tilde{f}_{j,t,1}(\omega_t) d\omega_t$ at $\psi_t = \bar{\psi} \neq -\infty$. Alternatively, we can interpret $H_y(\bar{\psi})$ from the perspective of distorted subjective beliefs of all DMs. At Amb.-SS, the amount of ambiguity $\bar{\psi}$ plays a non-trivial role in the sense that the subjective belief about aggregate productivity is distorted in the mean. Such a mean distortion must be respected when we evaluate the Amb.-SS, leading to a non-zero term $H_y(\bar{\psi})$. Similar arguments can be found in Ilut and Schneider (2014) and Ilut and Saijo (2021) in the context of "worst case" belief due to multiple prior preferences.

To arrive at these equilibrium conditions, we use the fact that, under the proposed policy rules, the following is true

$$\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}[\hat{y}_t]\tilde{f}_{j,t,1}(\omega_t)d\omega_t$$
$$= \left[1 - \left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right)\right]\kappa_{ya_j}(\psi_t)a_{j,t} + H_y(\widehat{\psi}_t, \lambda) - H_y(\bar{\psi}, \lambda),$$

where $H_y(\psi_t, \lambda)$ being given by

$$H_y(\psi_t, \lambda) = \kappa_{ya_j}(\psi_t)\left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right) g_\mu(\psi_t, \lambda). \tag{A.5}$$

In what follows, we first demonstrate that there exists a unique Amb.-SS. Upon establishing the uniqueness of Amb.-SS, we then move on to prove that for any given amount of ambiguity ψ_t, there exists a unique $\kappa_{ya_j}(\psi_t)$, which stands for the use of private information. Finally, the existence and uniqueness of $\hat{h}_y$ would be straightforward given all the results we have established, which completes the whole proof.

Amb.-SS can be characterized by the following equations:

$$\left(\frac{1+\varepsilon}{1-\alpha}\right)\bar{\kappa}_{ya_j} = \left(\frac{1+\varepsilon}{1-\alpha}\right) - \left(\frac{1}{\theta} - 1\right)\bar{\kappa}_{ya_j}\left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\bar{\psi}}}\right)$$

$$\left(\frac{1+\varepsilon}{1-\alpha}\right)\bar{h}_y = -\left(\frac{1}{\theta} - 1\right)\bar{\kappa}_{ya_j}\left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\bar{\psi}}}\right)\lambda\bar{\kappa}_{J\omega}e^{\bar{\psi}}$$

$$\bar{\kappa}_{J\omega} = \bar{\kappa}_{ya_j}.$$

The existence and uniqueness of the pair $(\bar{\kappa}_{ya_j}, \bar{h}_y)$ can be directly proved.

After proving the uniqueness of Amb.-SS, we proceed to prove that the use of private information $\kappa_{ya_j}(\psi_t)$ is unique. The use of private information κ_{ya_j} is determined by (A.3). Denote the gap between LHS and RHS of (A.3) as $f(\kappa_{ya_j}; \psi_t)$ such that

$$f(\kappa_{ya_j}; \psi_t) \equiv \left(\frac{1+\varepsilon}{1-\alpha}\right)\kappa_{ya_j} - \left(\frac{1+\varepsilon}{1-\alpha}\right) + \left(\frac{1}{\theta} - 1\right)\left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right)\kappa_{ya_j}. \tag{A.6}$$

It is then straightforward to show that the unique solution to $f(\kappa_{ya_j}) = 0$ is

$$\kappa_{ya_j} = \frac{\left(\frac{1+\varepsilon}{1-\alpha}\right)}{\left(\frac{1+\varepsilon}{1-\alpha}\right) + \left(\frac{1}{\theta} - 1\right)\left(\frac{\sigma_\iota^2}{\sigma_\xi^2+\sigma_\iota^2+e^{\psi_t}}\right)}.$$

Finally, in the last step of the proof, it is straightforward to demonstrate the existence and uniqueness for $\hat{h}_y$ from (A.4) given the existence and uniqueness for Amb.-SS and κ_{ya_j}. □

Proof of Proposition 2

It directly follows the comparison between Proof of Proposition 1 and the solution for the beauty contest identified in the proposition. The comparative static analysis of g_μ can be proved by showing that $\kappa_{J\omega}$ is increasing in ψ_t. □

Proof of Proposition 3

It can be shown that (A.6) has the following properties regarding its partial derivatives evaluated at the equilibrium, that is, $f(\kappa_{ya_j}; \psi_t) = 0$:

$$(1)\quad \frac{\partial f(\kappa_{ya_j}; \psi_t)}{\partial \kappa_{ya_j}}|_{f(\kappa_{ya_j};\psi_t)=0} > 0\,,$$

$$(2)\quad \frac{\partial f(\kappa_{ya_j}; \psi_t)}{\partial \psi_t}|_{f(\kappa_{ya_j};\psi_t)=0} < 0$$

which are all straightforward following the fact that $1/\theta - 1 > 0$. Therefore, $\kappa_{ya_j}(\psi_t)$ is increasing in ψ_t.

Furthermore, it can be shown that $\kappa_{J\omega}$ is increasing in ψ_t since it is increasing in κ_{ya_j}, which is an increasing function of ψ_t. Moreover, note that we can transform (A.5) into

$$H_y(\psi_t, \lambda) = -\kappa_{ya_j}(\psi_t)\left(\frac{\sigma_\iota^2}{\sigma_\xi^2 + \sigma_\iota^2 + e^{\psi_t}}\right)e^{\psi_t}\lambda\kappa_{J\omega}.$$

It is then straightforward to show that $H_y(\psi_t, \lambda)$ must be decreasing in ψ_t. Combined with (A.4), we can prove that $\hat{h}_y(\psi_t, \lambda)$ is decreasing in ψ_t. □

References

Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Lindé (2011). "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." *Review of Economic Dynamics*, 14, 225–247.

Angeletos, George-Marios, Fabrice Collard, and Harris Dellas (2018). "Quantifying Confidence." *Econometrica*, 86, 1689–1726.

Angeletos, George-Marios, Fabrice Collard, and Harris Dellas (2020). "Business-Cycle Anatomy." *American Economic Review*, 110(10), 3030–3070.
Angeletos, George-Marios, Luigi Iovino, and Jennifer La'O (2016). "Real Rigidity, Nominal Rigidity, and the Social Value of Information." *American Economic Review*, 106(1), 200–227.
Angeletos, George-Marios and Jennifer La'O (2009). *Noisy Business Cycles*. NBER Macroeconomics.
Angeletos, George-Marios and Jennifer La'O (2013). "Sentiments." *Econometrica*, 81, 739–779.
Angeletos, George-Marios and Chen Lian (2021). "Confidence and the Propagation of Demand Shocks." *Review of Economic Studies*, 89, 1085–1119.
Angeletos, George-Marios and Alessandro Pavan (2007). "Efficient Use of Information and Social Value of Information." *Econometrica*, 75, 1103–1142.
Arellano, Cristina, Yan Bai, and Patrick Kehoe (2018). "Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy*, 127, 2049–2103.
Baqaee, David Rezza (2020). "Asymmetric Inflation Expectations, Downward Rigidity of Wages, and Asymmetric Business Cycles." *Journal of Monetary Economics*, 114, 174–193.
Barsky, Robert B. and Eric R. Sims (2012). "Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence." *American Economic Review*, 102(4), 1343–1377.
Beaudry, Paul and Franck Portier (2004). "An Exploration into Pigou's Theory of Cycles." *Journal of Monetary Economics*, 51, 1183–1216.
Beaudry, Paul and Franck Portier (2006). "Stock Prices, News, and Economic Fluctuations." *The American Economic Review*, 96(4), 1293–1307.
Benhabib, Jess, Pengfei Wang, and Yi Wen (2015). "Sentiments and Aggregate Demand Fluctuations." *Econometrica*, 83, 549–585.
Bhandari, Anmol, Jaroslav Borovicka, and Paul Ho (2019). "Survey Data and Subjective Beliefs in Business Cycle Models." FRB Richmond Working Paper No. 19-14.
Bianchi, Francesco, Cosmin L. Ilut, and Martin Schneider (2017). "Uncertainty Shocks, Asset Supply and Pricing over the Business Cycle." *The Review of Economic Studies*, 85, 810–854.
Bidder, R.M. and M.E. Smith (2012). "Robust Animal Spirits." *Journal of Monetary Economics*, 59, 738–750.
Blanchard, Olivier J., Jean-Paul L'Huillier, and Guido Lorenzoni (2013). "News, Noise, and Fluctuations: An Empirical Exploration." *American Economic Review*, 103(7), 3045–3070.
Blanchard, Olivier Jean and Danny Quah (1989). "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review*, 79(4), 655–673.
Bloom, Nicholas (2009). "The Impact of Uncertainty Shocks." *Econometrica*, 77, 623–685.
Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry (2018). "Really Uncertain Business Cycles." *Econometrica*, 86, 1031–1065.
Chen, Heng, Yang K. Lu, and Wing Suen (2016). "The Power of Whispers: A Theory of Rumor, Communication, and Revolution*." *International Economic Review*, 57, 89–116.
Chen, Heng, Guangyu Pei, Qian Xin, and Xu Li (2022). "Heterogeneous Overreaction in Expectation Formation: Evidence and Theory." The Chinese University of Hong Kong Working paper.
Chen, Heng and Wing Suen (2016). "Falling Dominoes: A Theory of Rare Events and Crisis Contagion." *American Economic Journal: Microeconomics*, 8, 228–255.
Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113, 1–45.
Cochrane, John H. (1994). "Shocks." *Carnegie-Rochester Conference Series on Public Policy*, 41, 295–364.
Collard, Fabrice, Sujoy Mukerji, Kevin Sheppard, and Jean-Marc Tallon (2018). "Ambiguity and the Historical Equity Premium." *Quantitative Economics*, 9, 945–993.
Epstein, Larry G. and Martin Schneider (2008). "Ambiguity, Information Quality, and Asset Pricing." *The Journal of Finance*, 63, 197–228.
Fernald, John (2014). "Productivity and Potential Output Before, During, and After the Great Recession." Working Paper No. 20248, NBER.
Gilboa, Itzhak and David Schmeidler (1989). "Maxmin Expected Utility with Non-Unique Prior." *Journal of Mathematical Economics*, 18, 141–153.

Hanany, Eran and Peter Klibanoff (2009). "Updating Ambiguity Averse Preferences." *The B.E. Journal of Theoretical Economics*, 9, 1–53.
Hansen, Lars Peter and Thomas J. Sargent (2001a). "Acknowledging Misspecification in Macroeconomic Theory." *Review of Economic Dynamics*, 4, 519–535.
Hansen, Lars Peter and Thomas J. Sargent (2001b). "Robust Control and Model Uncertainty." *American Economic Review*, 91(2), 60–66.
Huo, Zhen and Naoki Takayama (2015). "Higher Order Beliefs, Confidence, and Business Cycles." Yale University Working paper.
Ilut, Cosmin (2012). "Ambiguity Aversion: Implications for the Uncovered Interest Rate Parity Puzzle." *American Economic Journal: Macroeconomics*, 4, 33–65.
Ilut, Cosmin, Matthias Kehrig, and Martin Schneider (2018). "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News." *Journal of Political Economy*, 126, 2011–2071.
Ilut, Cosmin and Hikaru Saijo (2021). "Learning, Confidence, and Business Cycles." *Journal of Monetary Economics*, 117, 354–376.
Ilut, Cosmin and Martin Schneider (2014). "Ambiguous Business Cycles." *American Economic Review*, 104(8), 2368–2399.
Jaimovich, Nir and Sergio Rebelo (2009). "Can News about the Future Drive the Business Cycle?" *American Economic Review*, 99(4), 1097–1118.
Ju, Nengjiu and Jianjun Miao (2012). "Ambiguity, Learning, and Asset Returns." *Econometrica*, 80, 559–591.
Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti (2010). "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57, 132–145.
Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji (2005). "A Smooth Model of Decision Making under Ambiguity." *Econometrica*, 73, 1849–1892.
Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji (2009). "Recursive Smooth Ambiguity Preferences." *Journal of Economic Theory*, 144, 930–976.
Lorenzoni, Guido (2009). "A Theory of Demand Shocks." *American Economic Review*, 99(5), 2050–2084.
Maccheroni, Fabio, Massimo Marinacci, and Doriana Ruffino (2013). "Alpha as Ambiguity: Robust Mean-Variance Portfolio Analysis." *Econometrica*, 81, 1075–1113.
Marinacci, Massimo (2015). "Model Uncertainty." *Journal of the European Economic Association*, 13, 1022–1100.
Morris, Stephen and Hyun Song Shin (2002). "Social Value of Public Information." *American Economic Review*, 92(5), 1521–1534.
Nimark, Kristoffer P. (2014). "Man-Bites-Dog Business Cycles." *American Economic Review*, 104(8), 2320–2367.
Peter, Hansen Lars and Massimo Marinacci (2016). "Ambiguity Aversion and Model Misspecification: An Economic Perspective." *Statistical Science*, 31, 511–515.
Stock, James H. and Mark W. Watson (1999). "Chapter 1 Business Cycle Fluctuations in us Macroeconomic Time Series." In *Handbook of Macroeconomics*. Elsevier, pp. 3–64.
Straub, Ludwig and Robert Ulbricht (2018). "Endogenous Uncertainty and Credit Crunches." MIT Working paper.
Zhang, Donghai (2022). "The Ambiguous Signaling Channel of Monetary Policy." University of Bonn Working paper.

Supplementary Data

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